Ibn ᶜAzzūz al-Qusanṭīnī's tables for computing planetary aspects

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Abstract

This paper presents a chapter of the fourteenth century *al-Zīj al-Muwāfiq* by Ibn al-ᶜAzzūz al-Qusanṭīnī (d 1354) containing the only known example of numerical tables designed for the computation of the astrological aspects following the most popular method for this practice in the Islamic area: that of the Single Hour Line. Ibn ᶜAzzūz presents two different procedures. The first one illustrates, according to the author, the method normally used in his time, mentioned by "Ptolemy and Hermes", and transmitted by Abu Maᶜshar. The second one is the technique that he uses for computing his tables for the latitude of Fes. The geometrical approach of both procedures is the same and what Ibn ᶜAzzūz offers with his tables is a computational tool that aims to improve the system avoiding certain errors produced by previous algorithms. Nevertheless, Ibn ᶜAzzūz's final belief is that the computation for the aspects must be performed on the ecliptic following the simple method — consisting of adding or subtracting the different aspects to the ecliptical longitude of the star or planet —, whereas the use of his tables is more appropriate for another astrological practice: the *tasyīr* or system of progressions.

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1. Introduction

The branch of astrology that deals with nativities aims to decipher the influence that certain elements of the celestial sphere exert on the life and personality of an individual according to their positions at the moment of birth. The computational component of this doctrine has three main concerns, all of them taken from the ancient Greek astrologers, namely: the division of houses (in Arabic *taswiyat al-buyūt*, which deals with the positions of these elements in relation to the local horizon), the theory of aspects or projection of rays (in Arabic *Maṭraḥ al-shuʿāʿāt*, concerned with the relative positions of these elements to each other) and the theory of progressions (in Arabic *tasyīr*, devoted to the determination of the moment at which a certain given position will take place between two of these elements)[1]. Since the geometrical interpretation of these questions requires mathematical definitions and calculations which are different from those applied to astronomy, the medieval astronomers and mathematicians had to provide the necessary tools for those practices, which were performed by means of arithmetical or trigonometric algorithms, numerical tables and analogical instruments. The investigation of these solutions, undertaken by historians of science in recent years, revealed the interest of sources with astrological contents for the history of applied mathematics and led to a classification of methods for the three aforementioned practices that correspond to a variety of geometrical approaches and also produce different results[2].

1 For these topics in the Greek astrological tradition, the standard reference is Bouché-Leclerq, *Astrologie*, 256-286 (houses), 165-179, 247-251 (aspects), 411-422 (progressions).

2 As a bibliographical list for the study of astrological materials, one can consider the following works, in chronological order, and the references that they contain: Kennedy & Krikorian, "Rays"; North, "Horoscopes" (containing a classification of seven methods found in ancient and medieval sources for the division of houses); Hogendijk, "Two Tables"; Kennedy, "Ibn Muʿādh" and "Houses" (research on the evidence of the methods for the division of houses of North's classification in Arabic and Persian sources, enlarging the classification with two new methods); North, "A reply"; Calvo, "Résolution graphique"; Hogendijk, "Progressions" (a complete classification and analysis of the methods applied in sources from the Islamic area to the three aforementioned astrological practices. Though an indispensable tool for the study of

The aim of this paper is to present a passage from a western Arabic source containing the only known preserved tables designed for the practice of projecting the rays according to what is called the Single Hour Line method. The text consists of a chapter of the fourteenth century *al-Zīj al-Muwāfiq* by Ibn al-ᶜAzzūz al-Qusanṭīnī (d 1354)[3], a work pertaining to an astronomical Maghribī tradition that combines the preservation of Andalusian materials with the result of new observations and correction of parameters using experimental methods. The passage of interest to us here is preserved in pages [428] - [437][4] of MS D 2461 of the Bibliothèque Générale de Rabat[5], one of the two known extant copies of the *zīj*[6], which contains a series of twelve tables, computed for the latitude of Fes, with the text of a chapter on the projection of rays in the margins of pages [428] - [435][7].

As a preliminary instrument for understanding this text and analysing

medieval astrology, this article remains unpublished. A photocopy of the typed paper was kindly made available to me by the author); Samsó & Berrani, "World astrology"; Casulleras, "Ibn Muᶜādh"; Hogendijk, "Applied Mathematics"; Samsó & Berrani, "al-Istijī"; Casulleras, "Aspectos" (Spanish review of the list of methods for the projection of rays established by Hogendijk, with special attention to procedures used in al-Andalus and sources which were not available at the time of Hogendijk's research).

3 On this author and his work see the studies by Samsó: "Ibn ᶜAzzūz" (with an outline of the *Muwāfiq Zīj*), "Horoscopes", "Astronomical observations", 166-167, "Zacut's Almanach", 82-83, "Maghribī Zijes", 94, all of them reprinted in Samsó, *Variorum*, 2007.

4 I use numbers in square brackets in order to refer to the corresponding indications of manuscript pages in my edition and translation of the text in appendixes 1 and 2; facsimile edition in appendix 4.

5 Djebbar was the first to call attention to the existence of the *Muwāfiq Zīj* in Djebbar, "Quelques élements", 67-68. Cf. Samsó, "Ibn ᶜAzzūz", 75-76 and n.7.

6 The other copy does not contain the tables for the projection of rays and is extant in manuscript 8772 of the Ḥasaniyya Library. See Samsó, "Ibn ᶜAzzūz", 76 and n.11, for details.

7 Cf. Samsó, "Ibn ᶜAzzūz", 95.

the accompanying tables, we need the definitions of some mathematical procedures for the projection of rays, all of them taken from the classification of methods established by Hogendijk in 1998[8], an essential tool for understanding the mathematical implications of the astrological practice. In his study of sources dealing with the computational techniques used by astrologers, Hogendijk found up to ten different methods for casting the rays. As a reference, in this introduction I give brief definitions for these procedures, as well as a list of the symbols used in the following pages, which may indicate either a value, a point of the celestial sphere or a function if followed by variables in parentheses. In order to try to put Ibn ᶜAzzūz's work in context, I also give the list of the other tables for the projection of rays known to me. It is assumed that the reader has some knowledge of the astronomical coordinate systems or access to an introduction to spherical astronomy[9].

Symbols used (in Greek alphabetical order)

- α_0 equatorial right ascension. α_0^{-1} is the inverse function of α_0, returning the ecliptical longitude λ corresponding to the equatorial degree represented by α_0. Example: given a point P on the ecliptic, with right ascension A on the equator, $\alpha_0(P) = A$, and $\alpha_0^{-1}(A) = P$.
- α_ϕ oblique ascension. Arc of the celestial equator which rises simultaneously with a given arc of the ecliptic at a place of latitude ϕ. Similarly, $\alpha_{-\phi}$ is for oblique descension, that is the oblique ascension for a horizon of latitude $-\phi$, and α_ξ is the oblique ascension at a horizon of latitude ξ. α_ϕ^{-1} and α_ξ^{-1} are respectively inverse functions of α_ϕ and α_ξ, returning the ecliptical longitudes λ corresponding to the equatorial degrees represented by α_ϕ and α_ξ.
- β ecliptical latitude.

8 Cf. Hogendijk, "Progressions", § 4. Readers of Spanish can also see Casulleras, "Aspectos".

9 Readers of Spanish can see, for example, Orús, Català & Nuñez, *Astronomía esférica* or Martín, *Astronomía*.

δ equatorial declination.

$\Delta\alpha$ ascensional difference. For an ecliptical degree λ, $\Delta\alpha(\lambda) = | \alpha_0(\lambda) - \alpha_\phi(\lambda) |$.

ϵ obliquity of the ecliptic.

λ ecliptical longitude: I use λ_1, λ_4, λ_7 and λ_{10} to indicate longitudes of the *cardines* (Arabic *watad*, pl. *awtād*) or intersections of the ecliptic with the local horizon and meridian, which are, respectively, the ascendent (first house), lower midheaven (fourth house), descendant (seventh house), and upper midheaven (tenth house).

ξ terrestrial latitude, other than ϕ, beeing $\phi > \xi > 0°$. $\overline{\xi}$ is the complement of ϕ: $90° - \xi$.

ϕ observer's terrestrial latitude. $\overline{\phi}$ is the co-latitude, this is $90° - \phi$.

Methods for projecting the rays

The Greek theory of the projection of rays is sometimes defined in terms of the astrologically significant elements emanating rays in certain directions. If one of these rays reaches another object at a particular meaningful angular distance, the two objects make an aspect. Depending on the arc or angle involved, this aspect can be a sextile (60°), a quartile (90°), a trine (120°), or an opposition (180°). Moreover, if the aspect is found in the sense of increasing ecliptical longitudes, it is a left ray, otherwise, it is right, with different astrological effects. Ideally, the complete set of aspects is defined by means of a regular hexagon, a square and an equilateral triangle, all of them inscribed in a great circle and with a common vertex at the position of the object that casts its rays (point *P* in Figure 1). The simplest procedure for finding the position of these rays or aspects consists of measuring the defining distances along the ecliptic but, as we shall see, most methods require the determination of these intervals along another great circle, thus demanding, first, a projection of an ecliptic point onto this circle and, later, a projection from a point of that cirle back to the ecliptic. Other approaches also take ecliptic latitudes into account.

Simple Ecliptical method ignoring latitude[10]

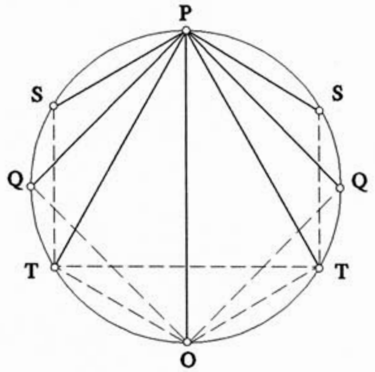

Figure 1

In this method the aspects are simply measured along the ecliptic. In Figure 1, *P* represents a planet casting its sextiles (*S*), trines (*T*), quadratures (*Q*) and opposition (*O*) directly on the ecliptical circle, such inscribing a series of regular polygons in it. If the planet has any ecliptical latitude (β), this can be ignored, and its longitude (λ) is taken as the common vertex of the polygons. This is the method favoured by Ibn ᶜAzzūz, though his tables are devised for the Single Hour Line method (see below).

[10] For details on the use of this method, see Hogendijk, "Progressions", § 4.1; Casulleras, "Aspectos", 31-32.

Ecliptical methods considering latitude[11]

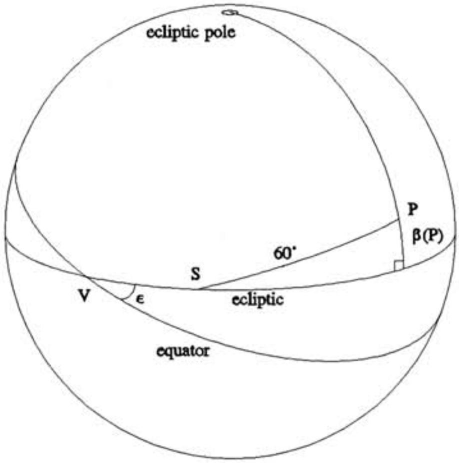

Figure 2

A more sophisticated theory takes as the origin for the rays the position of the planet without ignoring its ecliptical latitude (β). Hogendijk found two solutions for this alternative, following previous work by Kennedy and Krikorian[12].

In the first one, the angular distances for the different aspects are taken along arcs of the great circle connecting the object emanating rays with the ecliptic. Figure 2[13] represents the left sextile of a planet at P, with

[11] See Hogendijk, "Progressions", § 4.1; Casulleras, "Aspectos", 33-34.

[12] Cf. Kennedy & Krikorian, "Rays", 5-7.

[13] For this and the following figures I use version 2.0 of the computer program *ESB* (*Electronic Spherical Blackboard*) developed by H. Mielgo, University of León.

latitude $\beta(P)$: arc PS is 60° long and the longitude of the aspect is $\lambda = VS$.

The second procedure considers the regular polygons defining the aspects inscribed in the great circle that passes through the position of the planet and crosses the ecliptic at 90°, in such a way that — for any object with latitude $\beta \neq 0°$ — only the two quartiles would be on the ecliptic, whereas the other aspects would have their own latitudes.

Single Position Semicircle method[14]

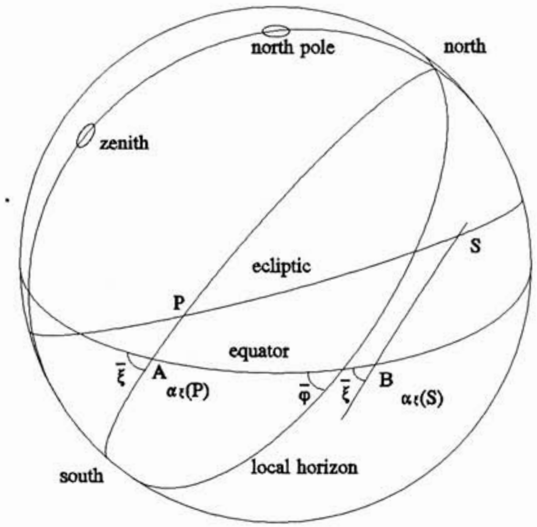

Figure 3

A position circle is a great circle crossing the North and South points on the local horizon, of latitude ϕ, and is equivalent to a horizon of a latitude ξ (beeing $\phi > \xi > 0°$) passing through a desired point of the celestial sphere. For this reason, some sources use the term *incident horizon* (*al-ufuq al-ḥādith*) for these circles, and are employed in various procedures for all the aforementioned astrological practices (division of houses,

[14] See Hogendijk, "Progressions", § 4.4; Casulleras, "Aspectos", 37-38.

projection of rays and *tasyīr*)[15]. The Single Position Semicircle method defines the aspects on the ecliptic using the semicircle through the North and South points of the local horizon and through the object that casts its rays. Figure 3 illustrates the case for a left sextile (S) of a planet P using this method. The procedure for determining any of the rays is as follows:

1) find the projection by means of the position circle that pass through the ecliptical position of the object emanating rays onto the celestial equator: point $A = \alpha_{\xi}(P)$ in Figure 3.

2) from this point (A), add or subtract the equatorial arc that corresponds to the degrees of the desired aspect, thus obtaining another equatorial point: point $B = \alpha_{\xi}(S)$ in Figure 3, with arc AB of 60°.

3) project this last point (B) back to the ecliptic using the same position circle as before. The resulting point corresponds to the ecliptical longitude of the desired aspect: point $S = \alpha_{\xi}^{-1}(B)$ in Figure 3.

Single Hour Line method[16]

This is the method for which Ibn ᶜAzzūz computed his tables and, in Hogendijk's opinion, it can be considered the standard method for the projection of rays in the Islamic area, given its presence in many sources. As an exact computation with position circles involves the use of functions and techniques of spherical trigonometry, since ancient times the authors developed arithmetical procedures that use seasonal hour lines as an approximation to those circles. The seasonal hours for the period of daylight are twelve equal divisions between sunrise and sunset; there are also twelve equal seasonal night hours between sunset and sunrise. Obviously, the length of these hours depends on the latitude of the place and on the season (or, more strictly, on the day) of the year. This system was the usual civil time-reckoning in ancient and medieval Greek, Roman, Islamic and European civilizations. An hour line corresponding to the end

15 Cf. Hogendijk, "Two Tables", 176-178; Kennedy, "Houses", 555, 557; Dorce, *Tāŷ al-Azyāŷ*, 63, 67-72. An early definition of this circles is found in the treatise on the construction of the astrolabe by al-Farghānī, probably written about 856-857, cf. Lorch, *Farghānī*, 5, 10, 60-63. I am grateful to Professor Samsó for this information.

16 See Hogendijk, "Progressions", § 4.5; Casulleras, "Aspectos", 38-41.

of one seasonal hour on the celestial sphere is a curve that links the positions of the sun at that given hour through the whole year[17].

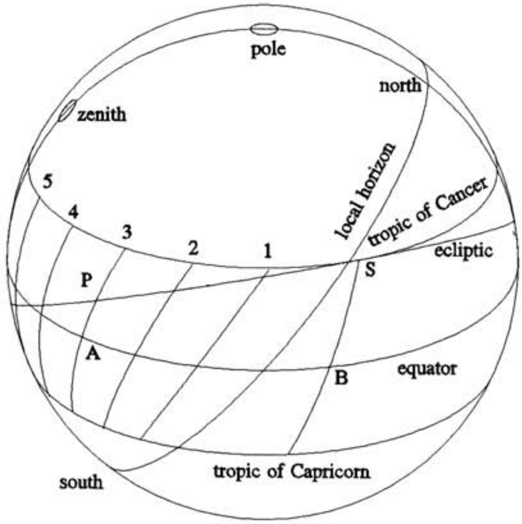

Figure 4

The Single Hour Line method uses the hour line that passes through the position of the object that casts the rays in the manner in which the position semicircle was used in the preceding method. Figure 4 shows the seasonal hour lines[18] at the eastern quadrant above the horizon for a latitude $\phi = 40°$. Point P is a planet on the ecliptic casting its left sextile at S according to this method and the hour line through S represents the

[17] Some mathematicians, at least from the end of the tenth century onwards, knew that the result is not normally an arc of a circle but a more complex curve. Nevertheless, for practical purposes, the hour lines on the plate of an astrolabe are obtained by dividing into equal parts the projections of the arcs of the equator and the tropics of Cancer and Capricorn between the local horizon and the meridian and joining each series of three divisions corresponding to the ordinal number of the same hour with arcs of circles. See Hogendijk, "Seasonal Hour Lines" for details.

[18] For simplicity, I also use arcs of circle as approximations to the true hour curves.

same hour line passing through *P* after 60° (arc *AB* on the equator) of rotation of the celestial sphere. As regards the computational solutions, the fundamental difference compared with the use of position circles is that the hour lines are regularly distributed along the equator and its parallel circles, round the axis of the earth and directly connected to the diurnal movement of the celestial sphere, allowing for approximate arithmetical formulae based on the application of proportions of time and simple interpolation coefficients.

Four Position Circles method[19]

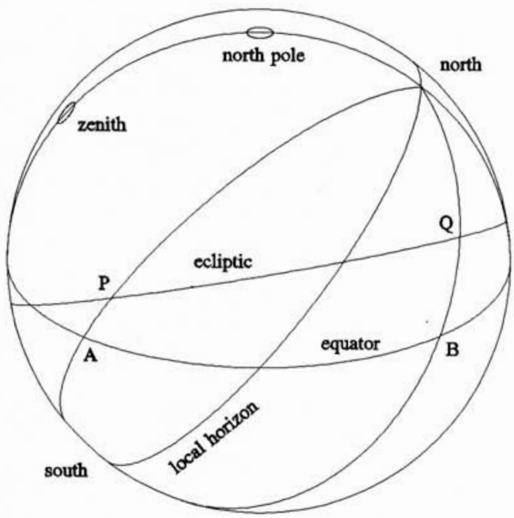

Figure 5

Unlike the Single Position Semicircle method, which uses a single semicircle for finding the whole system of aspects, in the Four Position

19 See Hogendijk, "Progressions", § 4.6; Casulleras, "Aspectos", 41-42.

Circles method the longitude of each different aspect is found by means of the position circle that crosses the point of the equator corresponding to the significant angular distance. Figure 5 represents the left quartile (*Q*) of a planet (*P*) using this method. The equatorial arc *AB* measures 90°. Arc *PA* belongs to a great circle passing through the ecliptic position of *P* and the north and south points of the local horizon, whereas arc *QB* belongs to another position circle passing through *B*. An exact solution by means of computation following this method requires the use of spherical trigonometric functions[20].

Seven Hour Lines method[21]

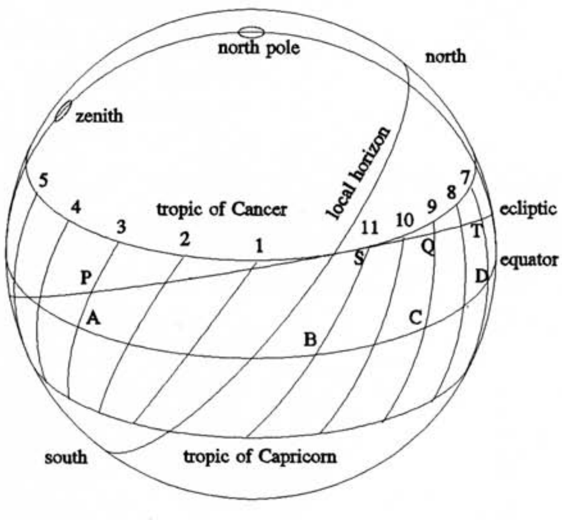

Figure 6

[20] On the origin of this method and its application to the division of houses and the projection of rays by the Andalusian mathematician Ibn Muᶜādh al-Jayyānī (d 1093), see Hogendijk, "Applied Mathematics".

[21] See Hogendijk, "Progressions", § 4.6; Casulleras, "Aspectos", 43.

Following this method, the rays of a star are at the intersection points of the ecliptic with the seasonal hour lines placed at a distance of four, six, eight or twelve seasonal hours of the seasonal hour line crossing the position of the star. Unlike the Single Hour Line method, this technique needs seven different hour lines for the whole set of aspects. Figure 6 shows the seasonal hour lines in the eastern hemisphere for a latitude $\phi = 40°$, and the left sextile (*S*), quartile (*Q*) and trine (*T*) of a planet (*P*) according to this procedure. The significant angular distances are placed on the equator: $AB = 60°$, $AC = 90°$ and $AD = 120°$.

Other methods

For the sake of completeness, we take into account the existence in medieval sources of other methods for projecting the rays, which will not be referred to here in connection with tables. These methods are:
- the Right Ascension method[22], which can be defined in the same terms as the Single Position Semicircle method, using a celestial meridian instead of a position semicircle.
- the Oblique Ascension method[23], in which the local horizon plays the role of the meridian in the Right Ascension method.
- the Standard Houses method[24], in which the different aspects of a planet are the longitudes of the third, fourth, fifth, seventh, ninth, tenth and eleventh astrological houses computed for the latitude corresponding to the position circle (or incident horizon) that passes through the planet and following what is known as the Standard method in North's classification of methods for the division of houses. In this method for the houses, given the right ascensions of the four cardines — $\alpha_0(\lambda_1)$, $\alpha_0(\lambda_4)$, $\alpha_0(\lambda_7)$ and $\alpha_0(\lambda_{10})$ —, one must trisect each of the resulting equatorial quadrants. The meridians that pass through these divisions determine the

22 See Hogendijk, "Progressions", § 4.2; Casulleras, "Aspectos", 34-36.

23 See Hogendijk, "Progressions", § 4.3; Casulleras, "Aspectos", 36-37.

24 See Hogendijk, "Progressions", § 3.7; Casulleras, "Aspectos", 42-43.

rest of the houses on the ecliptic[25].
- there is also evidence of astrolabes devised for the projection of rays using the prime vertical circle[26] for measuring the significant angular distances, although the textual description of this kind of procedure has not been found in the known sources[27].
- finally, there is a procedure for the stars with ecliptical latitude (β) described by ᶜAlī b. Abī-l-Rijāl (d ca 1048) which consists of projecting the star onto the ecliptic using the parallel of the equator that passes through the position of the star. The whole method for finding the aspects is not described in the text but there are indications in the same chapter that the Single Hour Line method could be used once the projection of the star onto the ecliptic is found[28].

Tables for projecting the rays

In addition to Ibn ᶜAzzūz's tables for the projection of rays, researchers have found some other sources containing tables for the same purpose. Hogendijk considers the tables contained in the revision of the *Zīj* of al-Khwārizmī (ca 830) by Maslama al-Majrīṭī (d 1007) adequate for an approximate computation of the Four Position Circles method, whereas, for those of al-Khwārizmī, based on the approximation to position circles

25 Cf. North, *Horoscopes*, 6, 46-47, 72; see also Kennedy, "Houses", 538-540.

26 This is the great circle that crosses the zenith and the east and west points of the local horizon.

27 See Hogendijk, "Applied Mathematics", 99 and n.3; Casulleras, "Aspectos", 43-44.

28 Cf. Casulleras, "Aspectos", 44-46; Ibn Abī-l-Rijāl, *Libro Conplido*, 173-178. In an unpublished work that she kindly allowed me to consult, Díaz-Fajardo identified a passage of Ibn Abī-l-Rijāl's chapter on the *tasyīr* and the projection of rays in the *al-Mughnī fī aḥkām al-nujūm* by Ibn Hibintā (Baghdad, ninth century); cf. Ibn Hibintā, *Mughnī*, 1: 131-134. Hogendijk relates the Ibn Hibintā's description to the Hour Line method for the *tasyīr*; cf. Hogendijk, "Progressions", § 3.1.4 and Ibn Hibintā, *Mughnī*, 1: 134-143.

using hour lines, he also suggests the Seven Hour Lines method[29]. Samsó reports a reference by al-Hāshimī (ca 890) on tables for projecting the rays that Hogendijk proposes to relate with those of al-Khwārizmī[30]. Finally, concerning the two ecliptical methods considering latitude, Kennedy and Krikorian observed that al-Bīrūnī gives, for both procedures, a description with a table in his *Qānūn* (Ghazna, Afganistan, ca 1030)[31].

The preceding list is rather short and it appears to be obviuosly incomplete if we consider that the same passage of al-Bīrūnī's *Qānūn* alludes to the existence of tables for the Single Hour Line method for the rays when dealing with its three possible means of resolution: "different people followed this method for casting the rays, using the computation, the tables or the instruments"[32]. Today, these tables are unknown but this poses the question of the originality and possible sources for Ibn ᶜAzzūz's tables. Our author only states that he "made tables for the projection of rays for the latitude of Fes" (page [431]), and explains the details of his computation, but he does not refer to the existence of previous sources containing similar tables. If we consider that the works by al-Bīrūnī and al-Hāshimī do not seem to have been known in the western Islamic area and that the tables of al-Khwārizmī's *Zīj* were unlikely to have been available after their replacement in the revision by al-Majrīṭī, we may suppose that Ibn ᶜAzzūz could have known the tables of al-Majrīṭī — which were computed for a different method — but also any other tables for the Single Hour Line method like those mentioned by al-Bīrūnī. It is important to remember that, in an arcane discipline like astrology, both suggesting and dismissing a possible chain of transmission are equally risky, lacking a direct quotation in a preserved source.

29 Cf. Hogendijk, "Progressions", § 4.6 and "Two Tables".

30 Cf. Samsó, "al-Bīrūnī", 600: n.47; Hogendijk, "Progressions", § 4.7: n.48; al-Hāshimī, *Reasons*, 186, 323-324.

31 al-Bīrūnī, *Qānūn*, 1385-1392. Cf. Kennedy & Krikorian, "Rays", 5-7 and Hogendijk, "Progressions", § 4.1.

32 Cf. al-Bīrūnī, *Qānūn*, 1385:
وقد قصد هذا الطريق من مطرح الشعاع أقوام من صنوف ما أخذ بالحساب وبالجداول وبالآلات ...

2. Title and colophon

Before going into detail, it is worth mentioning what seems to be a disagreement between the title of Ibn ᶜAzzūz's text and its contents. As stated above, the manuscript contains tables for the projection of rays at the latitude of Fes and a chapter on this same practice written in the margins. Nevertheless, the title reads "on knowing the projection of rays, the *tasyīr* of the stars and the division of houses", thus indicating that the three astrological practices are to be dealt with. Besides the obvious interpretation that the text may be truncated and the passages for the *tasyīr* and the houses have been lost, another explanation is also possible if we turn to the end of the text. As a colophon, in the last paragraph Ibn ᶜAzzūz says more or less that the aspects must be measured along the ecliptic following the simple method, and goes on to assert that "the fruit of these tables is in the *tasyīr* of the stars among themselves and in [determining] the value of the [arc of] *tasyīr* of one specific degree towards another degree which is known by the *tasyīr* ...", giving a clear sense to the reference to the *tasyīr* in the title, and connecting with a large tradition, attested at least from the time of Ptolemy (ca 150) and followed by most Islamic astronomers, which uses what Hogendijk calls the Hour Line method for the *tasyīr*, a procedure anologous to the Single Hour Line method for the rays described above[33]. It is more difficult to find a relationship between the tables and the division of houses: the text does not contain a single reference to this doctrine. I can only conjecture that Ibn ᶜAzzūz had in mind the possibility of using the tables for the division of houses following the Hour Lines method[34], an application that can be performed operating with their values in a suitable manner[35].

33 Cf. Hogendijk, "Progressions", § 3.1.4.

34 In this method, the houses are determined by the intersections of the ecliptic with the even seasonal hour lines. Cf. North, *Horoscopes*, 20-27; Hogendijk, "Progressions", 5.3.

35 This point merits further investigation. Samsó observed that a set of tables for the division of houses is preserved in a previous part (pages [366] - [377]) of the same work. Cf. Samsó, "Ibn ᶜAzzūz", 95.

3. The method of "Ptolemy and Hermes"

The chapter on the projection of rays attached to the tables begins (page [428]) with the exposition of a procedure presented as "what was mentioned by Ptolemy and Hermes" and transmitted by Abū Maᶜshar (d 886) from Ptolemy. As Hogendijk remarks[36], the attribution of methods to Ptolemy and Hermes is usual in medieval times but little evidence is actually found in the works of the Ptolemaic and Hermetic traditions. Indeed, only the Hour Line method for the *tasyīr* seems correctly attributed to Ptolemy, because it is presented in the *Tetrabiblos* as an approximation to the Position Semicircle method[37], but no single procedure for the projection of rays or the division of houses is included in this book. In the eleventh century, when explaining a computation for the Single Hour Line Method for projecting the rays, al-Bīrūnī observed in his *Qānūn* that the method is incorrectly attributed to Ptolemy but derived from his method of *tasyīr*[38], and the Alphonsine thirteenth century *Libro Segundo de las Armellas* expresses similar doubts on the authorship of the methods attributed to Ptolemy[39]. The case of Hermes is even more flagrant because the preserved astrological works related to him[40] do not justify any of the attributions of methods to this mythical author. Hogendijk's conclusion, based on the exploration of many texts, is that the attribution of a method to Ptolemy normally means that this method uses hour lines, whereas the attribution to Hermes implies the use of position circles or semicircles. The reference to a method of "Ptolemy and Hermes", repeated by Ibn ᶜAzzūz when dealing with the inaccuracies

36 Cf. Hogendijk, "Progressions", § 6.1.

37 This is an anologous procedure to the Single Position Semicircle method for the rays described above. Cf. Ptolemy, *Tetrabiblos*, 291; Hogendijk, "Progressions", §§ 3.1.3, 3.1.4, 6.1.

38 Cf. Hogendijk, "Progressions", § 4.5. Hogendijk gives the references to al-Bīrūnī, *Qānūn*, 2: 1377 (line 14), 1378 (line 4) and 1394.

39 Cf. Rico *Libros*, 2: 68; J. Casulleras, "Aspectos", 39-40.

40 Cf. Sezgin, *GAS*, 7: 50-58.

of the procedure in the following pages ([430] - [431]), is certainly peculiar and raises the question of a possible late identification of the procedures attributed to both authors or, at least, gives evidence that the attributions of methods to authors were not always consistent. Another related observation is found in the same work by al-Bīrūnī mentioned above: soon after stating that the Single Hour Line Method for the rays is erroneously attributed to Ptolemy, al-Bīrūnī goes further and declares that the method of Ptolemy for the rays needs position circles instead of hour lines[41].

The description of the method is presented by Ibn ᶜAzzūz as a quotation from Abū Maᶜshar but its development is excessively brief and confusing. Besides the use of a certain terminology, which is common to the descriptions of this kind of procedures, we cannot establish any textual relationship between Ibn ᶜAzzūz's report and the corresponding passage of the *al-Mudkhal al-Kabīr ilà ᶜilm aḥkām al-nujūm* as it is preserved[42]. Moreover, the procedure described by Ibn ᶜAzzūz agrees only partially with his purported source and, with some modifications which are needed in order to make the text a cohesive whole, it would match very well the computational procedure explained by al-Bīrūnī in his *Qānūn*[43]. The procedure presented by Ibn ᶜAzzūz (in [428] - [430]) under the name of Abū Maᶜshar is as follows:

First, we need to find "the ascendent and the tenth [house]". Then, for a

[41] The relevant passage is in al-Bīrūnī, *Qānūn*, at the beginning of page 1379, dealing with the case of the projection of the rays of a star that is found in an intermediate position, between the local meridian and the horizon: "... the best circles [...] are those that pass through the two intersections of the meridian with the horizon. These and their opposite [i.e. semicircles and their complements] are the circles to be used in Ptolemy's method".
... وأحقّ الدوائر [...] وهي التي تجتاز على تقاطعي فلك نصف النهار والأفق فهي ونظائرها هي الدوائر المقصودة للعمل في طريق بطلميوس .

[42] Lemay, *Abū Maᶜšhar*, 3: 549-550 (ed. Sezgin, 7: 408-410).

[43] Cf. al-Bīrūnī, *Qānūn*, 1377-1385.

star S and for *aspect* $= \{\pm 60°, \pm 90°, \pm 120°\}$[44], we find

1) $r_1(S) = \alpha_0^{-1}(\alpha_0(S) + aspect)$, and
2) $r_2(S) = \alpha_\phi^{-1}(\alpha_\phi(S) + aspect)$.

In the second case, the text is extremely concise: "do the same [as before] using oblique ascensions" names in the descriptions of Abū Maᶜshar and al-Bīrūnī. This last author takes into account the possibility of using oblique descensions — $\alpha_{-\phi}(S)$ instead of $\alpha_\phi(S)$ — when the star is in the western celestial hemisphere but his definition of $\alpha_{-\phi}$ as it is preserved is erroneous[45]. Ibn ᶜAzzūz continues with the observation that if the two

[44] Note that the manuscript text does not mention the subtraction of aspects. I have added this to the translation in order to have indications for the left and right aspects.

[45] In *Qānūn*, 1379, al-Bīrūnī states that:
"... it is known that right ascensions indicate the presence of the star on the diurnal or nocturnal meridian, that oblique ascensions indicate its presence on the eastern horizon, and that oblique descensions, which equal the [oblique] ascensions of the nadir, you indicate with them its presence in the western horizon."
فمعلوم أنّ مطالع خطّ الاستواء يدلّ عليها منذ كون الكوكب على فلك نصف النهار أو الليل وأنّ مطالع البلد يدلّ عليها عند كونه على أفق المشرق وأنّ مغارب البلد المساوية لمطالع النظير تدلّ عليها عند كونه على أفق المغرب .
The last case refers to the fact that for an ecliptical degree S: $\alpha_{-\phi}(S) = \alpha_\phi(S - 180°) - 180°$ (modulo 360°). Cf. Hogendijk, "Two Tables", 178.

On page 1380, we find that:
"... if the star is on the degree of the descendent, we perform with the degree of the ascendent, with oblique ascensions, the preceding [operations]. Then, we add to each of the results of the transformation [of coordinates] 180 degrees, and results in the projections of these rays."
وان كان الكوكب في درجة الغارب عملنا بدرجة الطالع في مطالع البلد ما تقدّم ثمّ زدنا على كلّ واحد مما يخرجه التقويس فيها مائة وثمانين درجة فينتهي الى مطارح تلك الشعاعات.
The "preceding [operations]" are the same as in the procedure described by Ibn ᶜAzzūz for finding r_1 and r_2, and al-Bīrūnī's text proposes, for a star of longitud S, to find the longitude of the desired *aspect* ($\pm 60°$, $\pm 90°$, $\pm 120°$) with $r_2(S) = \alpha_\phi^{-1}[\alpha_\phi(S \pm 180°) + aspect] \pm 180°$, which is erroneous. The correct computation, equivalent to the use of oblique descensions: $r_2(S) = \alpha_\phi^{-1}[\alpha_{-\phi}(S) + aspect]$, is $r_2(S) = \alpha_\phi^{-1}[\alpha_\phi(S \pm 180°) + aspect \pm 180°]$, that is, the transformation of coordinates must be done at the end of the operation:

Finally, on page 1383, we find that:
"... if [the star] is in the descendent hemisphere, which contains from the fourth [house], to

variables coincide in "a single minute", this will be the *equalized ray* of the star, that is the position of the aspect we were looking for. The same observation, but expressed diffrently, appears in Abū Maᶜshar's text[46]. The question is obvious: the two variables can only coincide if the star has no declination, $\delta(S) = 0°$, or the place has no latitude, $\phi = 0°$. If the variables diverge, one must obtain the difference (*ikhtilāf*) between the two rays by subtracting the closer one from the further one, counting on the ecliptic from the position of the star:

ikhtilāf $= |r_1(S) - r_2(S)|$, with suitable variations of this equation for the case that the origin of longitudes (Aries 0°) is between $r_1(S)$ and $r_2(S)$.

Next, the text instructs us how to find the distance (*buᶜd*) in seasonal hours from the star to the local meridian using right ascensions. The term *buᶜd* has different meanings depending on the authors. In this case, the concept is the same used by al-Bīrūnī[47] — expressed in degrees instead of seasonal hours — but it is different from that used by Abū Maᶜshar,

the descendent, to the tenth [house], we take the oblique ascensions of the nadir of the degree and do with them the same as we did with its ascensions [i.e. the oblique ascensions of the degree itself], we add 180 degrees to the results of the transformations and this gives the second ray."
ان كانت في النصف المنحدر الذي من الرابع الى الغارب الى العاشر أخذنا مطالع نظير درجته في البلد وفعلنا بها ما فعلنا بمطالعها فما خرج من التقويس فيها زدنا عليه مائة وثمانين درجة فينتهي الى الشعاع الثاني .
As stated above, the correct operation consists of taking the oblique ascensions of the nadir of the degree, adding 180° to the result plus the value of the desired aspect, and performing the transformation to ecliptical longitudes using oblique ascensions at the end of the whole process.

46 Cf. Lemay, *Abū Maᶜšhar*, 3: 550.
"... if both the right ascensions and the ascensions of the city [i.e. oblique ascensions] take place on one same degree and minute, the ray of the star is in this degree and minute."
فإن وقع مطالع الفلك المستقيم ومطالع المدينة كليهما بحيال جزء واحد ودقيقة واحدة، فشعاع الكوكب في تلك الدرجة والدقيقة .

47 Al-Bīrūnī also devoted a chapter to the explanation of the different meanings of this word. Cf. al-Bīrūnī, *Qānūn*, 1375-1377.

who finds the distance (*buʿd*) to the first cardine[48] towards which the star is travelling following the diurnal movement. Thus, according to Ibn ʿAzzūz, one has to find first the distance in degrees, using

$$bu^cd = |\alpha_0(S) - \alpha_0(\lambda_{10})| \text{ , if the star is above the horizon, and}$$
$$bu^cd = |\alpha_0(S) - \alpha_0(\lambda_4)| \text{ , if the star is below the horizon,}$$

and transform the resulting degrees into seasonal hours. At this point, the text is confusing and probably incomplete. The passage proposes two possible cases: the first one, having the star "in the southeastern quadrant [of the sphere] or in its nadir, which is the northwestern quadrant", and the second one, with the star "in the southwestern quadrant or in its nadir, which is the northeastern quadrant". Following the literal instructions of the text, in the first case, one has to divide the distance (*buʿd*) by the length of a seasonal diurnal hour of the degree of the star, whereas in the second case, one must divide it by the length of a seasonal nocturnal hour. In their normal sense, the words "western" and "eastern" refer to the position of a star with respect to the local meridian, whereas the distinction between "northern" and "southern" is frequently used to indicate the sign of the equatorial declination (δ) of the star. Nevertheless, both elements seem to be extraneous to the problem of converting the distance found in degrees into seasonal hours. One may conjecture that the text is corrupt and that the words "eastern" and "western" are traces of a lost complementary instruction on how to find the distance taking into account on which side of the local meridian the star is. As for the terms "northern" and "southern", they may have belonged to another lost description on how to find the length of a seasonal hour, a computation which certainly involves the sign of the equatorial declination.

Bearing in mind the expressed idea of finding the hours of distance from the star to the local meridian and the fact that we also find references to the hours of distance to the cardine ([429] - [430]), we may assume that, if the text were complete, it would prescribe transformation into seasonal hours of the smallest distance in degrees, measured on the equator, between the star and the local meridian. This operation is simply

[48] For this term, see in the introduction above the paragraph dealing with the Standard Houses method for the projection of rays.

performed by dividing the distance in degrees by the length of a diurnal or nocturnal seasonal hour, depending on whether the star is above or below the horizon.

Next, we find what the text calls equation (*ta*ᶜ*dīl*):

$$ta^{c}d\bar{\imath}l = bu^{c}d \text{ (in seasonal hours)} \times ikhtil\bar{a}f \,/\, 6 \,.$$

This definition is equivalent to that of the *ta*ᶜ*dīl* used by al-Bīrūnī[49], the only changes being the use of the *bu*ᶜ*d* in degrees and the use, as a divider, of the semidiurnal or seminocturnal arc of the star, depending on whether it is above or below the horizon, thus obtaining the same result.

The last step is to apply the *ta*ᶜ*dīl* to r_1, in a suitable way depending on whether r_1 or r_2 is greater, in order to obtain what the text calls an *equalized ray* (r_e). Thus

$$r_e(S) = r_1(S) + ta^{c}d\bar{\imath}l \text{ , if } r_1 < r_2 \text{ , and}$$
$$r_e(S) = r_1(S) - ta^{c}d\bar{\imath}l \text{ , if } r_1 > r_2 \,.$$

The application of the *ta*ᶜ*dīl* to obtain the final result also coincides with what al-Bīrūnī prescribes[50]. Consequently, according to my interpretation of Ibn ᶜAzzūz's text, the two procedures are equivalent with the exception that, as I said, al-Bīrūnī considers the use of oblique descensions for finding r_2 when the star is in the western half of the celestial sphere, whereas this possibility is not considered in the description reported by Ibn ᶜAzzūz, or in the preserved version of Abū Maᶜshar's *Mudkhal*. Moreover, this last text also uses a different concept of *bu*ᶜ*d*, as we have seen, and a different final application of the *ta*ᶜ*dīl*, for it regards the double possibility of having a right aspect, in which case the application is equal to that reported by Ibn ᶜAzzūz, or a left aspect, in which case one must invert the sign of the equation.

Finally, Ibn ᶜAzzūz declares (page [430]) that this "is the method of the

49 Cf. al-Bīrūnī, *Qānūn*, 1384. Ibn Muᶜādh al-Jayyānī uses the same expression but for a different system of computation for the rays, which corresponds to the Seven Hour Lines method: cf. Casulleras, "Ibn Muᶜādh", 400-402.

50 Cf. al-Bīrūnī, *Qānūn*, 1384.

wise people in our days". Nevertheless, the procedure has "some approximation if the star is in the two eastern quadrants, and a great flaw if the star is in the two western quadrants ... ". It is the lack of a correct use of oblique descensions in the calculations that generates these errors and the Ibn ᶜAzzūz's criticism.

4. The method of Ibn ᶜAzzūz's tables

The second part of Ibn ᶜAzzūz's text ([431] - [435]) is devoted to the structure of his tables "for the projection [...] of the rays for the latitude of Fes" and the way they should be used. Both the computation of the tables and the instructions for their use perfectly agree with the idea of using hour lines for defining the aspects of a star or planet. The tables are intended to contain the degrees corresponding to the points of the equator resulting from projecting onto this circle the ecliptical degrees using hour lines. The different columns give:

column 1: arguments, ecliptical degrees, each table for a sign and its nadir.
column 2: right ascensions of the arguments, α_0(column 1), for an obliquity of the ecliptic of $\epsilon = 23;33°$, not stated in the text but used by Ibn ᶜAzzūz in other tables of the same *Zīj*[51];
columns 3 to 7 (headers *alif* ... *hā'* = 1 ... 5): equatorial degrees of the projections of the arguments using the hour line corresponding to the header of the column;
column 8 (header *wāw* = 6): oblique ascensions of the argument, α_ϕ(column 1), for $\phi = 33;40$, not stated but used by Ibn ᶜAzzūz for the latitude of Fes elsewhere[52];
column 9: length of a seasonal hour (*azmān al-sāᶜāt*) for the degree of the argument.

The text ([432] - [434]) also explains how Ibn ᶜAzzūz computed the

[51] Cf. J. Samsó, "Ibn ᶜAzzūz", 93.

[52] Samsó found this table and the one in column 9 in a previous part of the manuscript, where the value for the latitude is given. Cf. Samsó, "Ibn ᶜAzzūz", 95.

tables. Take 1/6 of the ascensional difference of each degree of the argument: $\Delta\alpha(\lambda)/6$, for $\lambda = \{1° \ldots 360°\}$, found by $1/6 \times$ (column 2 − column 8)[53]. The columns for hours 1 to 5 are found by subtracting, respectively, 1/6, 2/6 .. 5/6 of the ascensional difference from the value for hour 0. Since Ibn ᶜAzzūz realized that there exists a symmetry between the ascensions of a sign using a given hour line and the descensions of the nadir of this sign using the hour line at the same distance with respect to the horizon in the western quadrant, he explains that he only wrote the values corresponding to the projections in the eastern (upper) quadrant of the celestial sphere. Though not explicitly expressed in the text, it is assumed that there also exists a symmetry between the ascensions of a sign using the hour lines at the same distance (measured on the celestial equator) above and below the horizon. The titles of the sub-tables "ascensions of the sign (Aries, etc.) and descensions of (the corresponding nadirs, this is Libra, etc.) in hours of distance with respect to the western horizon" are rather confusing. The text, however, is more explicit and one must understand that the digit (*alif*, *bā'*, etc.) on the header of each column indicates the distance from the meridian to the eastern or western horizon depending on whether we are using the sign or its nadir. Thus, column 2 (hour 0) corresponds to α_0(column 1), degree at midheaven or at lower midheaven, whereas column 8 (hour 6) corresponds to α_ϕ(column 1), degree on the eastern horizon or $\alpha_{-\phi}$(column 1), nadir of the degree on the western horizon. The values in column 9 allow for transformation into seasonal hours of the distance in degrees from the argument to the upper meridian, in order to enter the columns with the adequate parameters.

The instructions on the use of the tables, on pages [434] - [435], are as follows:

1) Given an ecliptical degree, λ, that corresponds to the star that projects its rays, find its distance to the horizon, measured on the

[53] In a context of absence of negative numbers, the text requires |column 2 (hour 0, α_0) − column 8 (hour 6, α_ϕ)| but the absolute operator can be obviated if we consider that, when using the absolute difference, Ibn ᶜAzzūz gives instructions on adding or subtracting it depending on the sign of the declination of the ecliptical degree involved: $\delta(\lambda)$.

celestial equator, using $|\alpha_0(\lambda_1) - \alpha_0(\lambda)|$ or $|\alpha_0(\lambda_7) - \alpha_0(\lambda)|$ depending on whether the star is, respectively, on the eastern or western celestial hemisphere. Divide this distance by the value of a diurnal or nocturnal seasonal hour for that degree, depending on whether it is above or below the horizon, thus obtaining the distance in hours.
2) If the star is in the eastern hemisphere, use the sub-table entitled "ascensions of ..." the sign of the star; otherwise, use the sub-table for the descensions of this sign. Enter the row corresponding to the given degree and the column corresponding to the distance in seasonal hours, now measured from the meridian (the text says from the horizon). The table gives the projection (ascensions or descensions) of the ecliptical degree onto the equator using the hour line for that hour.
3) To this number add or subtract the value for the desired aspect. Using the table in reverse mode, search the resulting quantity (corresponding to the projection of the ecliptical degree of the ray) among the tabular values in the column for the same hour through the following or preceding pages. The argument of this row is the longitude of the ray. If we also have fractions of hour, the text describes a linear interpolation method.

The above is a report of the manuscript's literal instructions. Nevertheless, the text omits two details that the user can easily deduce. On the one hand, when dealing with stars in the western hemisphere, using the table for the descensions of the degree of the star we need to use the nadir of the star because we are indeed operating with the degree of the opposition. The symmetries considered in the tables are based on the fact that the ascensions of a sign equal the descensions of its nadir in terms of the arc of the celestial equator involved (this can be simply verified by rotating the rete of a standard astrolabe), but this observation does not consider the origin of coordinates, which will cause a difference of 180° between the descensions of a degree and the descensions of its nadir. Consequently, on the other hand, when applying the corresponding amounts of the different aspects {-60°, -90°, -120°, ±180°, +120°, +90°, +60°} to the number of descensions found in the table, we obtain the equatorial points for the aspects of the opposition and we have to consider the points diametrically opposed to them (cf. Figure 1), thus changing the above sequence by {+120°, +90°, +60°, ±180°, -60°, -

90°, -120°}.

After these specifications, in order to illustrate the use of the tables (edited and recomputed[54] in appendix 3), I give numerical examples of it, considering possible positions of a star in the four quadrants of the celestial sphere determined by the local horizon and meridian planes. To allow comparison with the use of the algorithm for the Single Hour Line method as described in section 3, I include the corresponding results obtained following Ibn ᶜAzzūz's report but taking into account the use of oblique descensions prescribed by al-Bīrūnī. Since the use of hour lines, as stated in the introduction, is sometimes intended to be an approximation to the use of position circles, I also give the results that produce the Single Position Semicircle method at the same situations[55]. Finally, I give the results obtained with the Simple Ecliptical method, remembering that this is Ibn ᶜAzzūz's favourite method. In all cases, I use the values $\epsilon = 23;33°$, $\phi = 33;40°$, and a longitude of the ascendent $\lambda_1 = 270°$ ($\lambda_{10} = 198;19°$, $\lambda_7 = 90;0°$, $\lambda_4 = 18;19°$).

[54] For the recomputation I follow the procedure described in the manuscript itself. Hogendijk noted that there is another relationship between the columns two through eight and column nine, namely: (column nine) $-$ 15° = (1/6) × the ascensional differenece = (minus) the constant difference between column $(n+1)$ and column n, for $n = 2, 3, 4, 5, 6, 7$. This makes it easy to check columns two to eight approximately.

[55] For the computations according to this method one can use the following procedure, for a star of longitude λ and an *aspect* = {60°, 90°, 120°, 180°}: (1) the first part of an algorithm developed by Ibn Muᶜādh al-Jayyānī for the Four Position Circles method gives the projection onto the equator of the star casting its rays using the position circle passing through the longitude λ of the star, corresponding to a horizon of latitude ξ: this is $\alpha_\xi(\lambda)$ (cf. point *A* in Figure 3); modern formulae for Ibn Muᶜādh's computations are found in Hogendijk, "Applied Mathematics", 96-97: his section corresponding to the *computation of point K* gives the solution to our problem in this step; (2) to this $\alpha_\xi(\lambda)$ we apply the value of the *aspect* in order to find another equatorial point, corresponding to the projection onto the equator of the position of the desired ray using the same position circle thus obtaining $\alpha_\xi' = \alpha_\xi(\lambda) \pm$ *aspect* (cf. point *B* in Figure 3); (3) we can obtain the underlying value for the latitude of ξ using $\tan \xi(\alpha_\xi, \lambda) = \sin[\, \alpha_\xi - \alpha_0(\lambda)\,] / \tan \delta(\lambda)$; the values for $\alpha_0(\lambda)$ and $\delta(\lambda)$ can be obtained, respectively, with $\sin \delta(\lambda) = \sin \lambda \cdot \sin \epsilon$ and $\tan \alpha_0(\lambda) = \cos \epsilon \cdot \tan \lambda$; (4) finally, the longitude *R* of the desired ray is found with the expression $\sin R(\alpha_\xi', \xi) = \sin \alpha_\xi' \cdot \cos \xi / \sin[\, \cos^{-1}(\cos \epsilon \cdot \sin \xi + \sin \epsilon \cdot \cos \xi \cdot \cos \alpha_\xi')\,]$ (cf. point *S* in Figure 3).

Example for the upper-eastern quadrant

λ = 219; 0°, $\alpha_0(\lambda_1)$ = 270; 0°, $\alpha_0(\lambda)$ = 216;35°, diurnal hour length = 13;20°, distance in seasonal hours = |270; 0° − 216;35°| / 13;20° = 4; 0 hours. We use the column for hour 2, the distance from the meridian. Tabular value for λ and hour 2 = 219;55°.

aspect	ascensions in hour 2	nearest tabular value	tabular λ	Single Hour Line method	Single Position Semicircle method	Simple Ecliptic method
-60°	159;55°	159;47°	160°	158; 9°	161; 6°	159; 0°
-90°	129;55°	129;25°	131°	129; 0°	132;48°	129; 0°
-120°	99;55°	99;47°	104°	101;39°	105;38°	99; 0°
180°	39;55°	39;41°	46°	45; 0°	47;49°	39; 0°
120°	339;55°	339;54°	336°	335;55°	336;18°	339; 0°
90°	309;55°	309;53°	303°	303;14°	302;47°	309; 0°
60°	279;55°	279;58°	274°	274;29°	273;34°	279; 0°

Example for the upper-western quadrant

λ = 153; 0°, $\alpha_0(\lambda_7)$ = 90; 0°, $\alpha_0(\lambda)$ = 154;58°, diurnal hour length = 16;11°, distance in seasonal hours = |90; 0° − 154;58°| / 16;11° = 4; 1 hours. We the use column for hour 2 and the nadir of λ = 153° + 180° = 333°. Tabular value for 333° and hour 2 = 337;18°. We also consider the correspondences on the celestial equator between the aspects of the degree and those of its opposition.

aspect	ascensions in hour 2	nearest tabular value	tabular λ	Single Hour Line method	Single Position Semicircle method	Simple Ecliptic method
(-60°) 120°	277;18°	277;49°	272°	270;41°	269; 8°	273; 0°
(-90°) 90°	247;18°	246;57°	244°	243;53°	242;30°	243; 0°
(-120°) 60°	217;18°	217;48°	217°	216;26°	215;58°	213; 0°
(180°) 0°	157;18°	157;45°	158°	153; 0°	153; 0°	153; 0°
(120°) -60°	97;18°	97;35°	102°	103;24°	107;26°	93; 0°
(90°) -90°	67;18°	67;16°	74°	76;24°	80;10°	63; 0°
(60°) -120°	37;18°	36;54°	43°	46;21°	49; 9°	33; 0°

Example for the lower-western quadrant

λ = 67; 0°, $\alpha_0(\lambda_7)$ = 90; 0°, $\alpha_0(\lambda)$ = 65; 9°, nocturnal hour length = 12;27°, distance in seasonal hours = |90; 0° − 65; 9°| / 12;27° = 2; 0 hours. We use the column for hour 4 and the nadir of λ = 67° + 180°

= 247° tabular value for 247° and hour 4 = 255;21°. We also consider the correspondences between the aspects.

aspect		ascensions in hour 4	nearest tabular value	tabular λ	Single Hour Line method	Single Position Semicircle method	Simple Ecliptic method
(-60°)	120°	195;21°	195;20°	194°	193; 7°	196; 6°	187; 0°
(-90°)	90°	165;21°	165;47°	167°	165;17°	170; 4°	157; 0°
(-120°)	60°	135;21°	135;38°	140°	138;18°	144;19°	127; 0°
(180°)	0°	75;21°	75;28°	87°	67; 0°	67; 0°	67; 0°
(120°)	-60°	15;21°	15;43°	21°	21;41°	26;38°	7; 0°
(90°)	-90°	345;21°	345;05°	340°	341; 4°	343;23°	337; 0°
(60°)	-120°	315;21°	315;21°	304°	304; 1°	304;23°	307; 0°

Example for the lower-eastern quadrant

$\lambda = 302;0°$, $\alpha_0(\lambda_1) = 270;0°$, $\alpha_0(\lambda) = 304;17°$, nocturnal hour length = 17;19°, distance in seasonal hours = $|270;0° - 304;17°| / 17;19°$ = 1;59 hours. We use the column for hour 4, the distance from the meridian. Tabular value for λ and hour 4 = 313;33°.

aspect	ascensions in hour 4	nearest tabular value	tabular λ	Single Hour Line method	Single Position Semicircle method	Simple Ecliptic method
-60°	253;33°	253; 2°	245°	245;26°	245;14°	242; 0°
-90°	223;33°	223;15°	219°	219;13°	219;55°	212; 0°
-120°	193;33°	193;10°	192°	192; 7°	194;12°	182; 0°
180°	133;33°	133;24°	138°	138; 0°	142;21°	122; 0°
120°	73;33°	73;21°	85°	86; 1°	90;54°	62; 0°
90°	43;33°	43;42°	55°	56;33°	60;59°	32; 0°
60°	13;33°	13;23°	18°	20;27°	23;28°	2; 0°

5. Conclusions

Besides the obvious conclusion that the attributions of methods to authors in astrology are not always reliable, we have also seen an example of how a single method can generate a variety of procedures by way of transmission through different authors. This is the case of the accounts for the Single Hour Line method for the projection of rays found in al-Bīrūnī, Abū Maᶜshar and the version of the latter in Ibn ᶜAzzūz. If the motivation for compiling his tables was the lack of precision of the procedure when dealing with stars in the western hemisphere, Ibn ᶜAzzūz could have spared himself this effort if he had known al-Bīrūnī's description for the same method, because this version takes into account oblique descensions when necessary. Nevertheless, thanks to the imperfection of the version available to Ibn ᶜAzzūz, we have the only known preserved case of a table for the Single Hour Line method for the rays and we can check the results that an astrologer could obtain using this tables.

The comparison with the data obtained with other methods leads to another conclusion: considering the differences found using one method or another, the choice of a satisfactory procedure is not a trivial question from an astrologer's point of view. Notwithstanding, for our interpretations, the problem is that there is not always evidence in the sources of the authors' awareness of subtleties like the degree of approximation reached using hour lines with respect to the use of position circles, nor of the differences using one method or another. Together with a tradition that distinguishes methods that use hour lines (attributed to Ptolemy) from those using position circles or semicircles (attributed to Hermes) there seems to be another tradition that understands the use of hour lines as an approximation to the use of position circles or semicircles. In this sense, it is difficult to assert whether these traditions emerged in parallel or were generated by confusion, and to what extent the astrologers were conscious of the fact that the two sets of systems belong to different geometric models. Moreover, Ibn ᶜAzzūz's proposal of performing the projection of rays following the Simple Ecliptic method and using his tables for the *tasyīr* gives a clear example of the application of a single method to different astrological practices without any theoretical justification and without heeding of the numerical consequences and astrological implications. This example contributes to strengthen the conviction expressed by North in 1996[56] that one of the causes for the cross-fertilization between the methods is the availability of a given instrument (in this case, a table) that one tends to use for different purposes.

[56] Cf. North, "A reply", 582.

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Appendix 1. Translation of the text attached to the tables

[428] Chapter on knowing the projection of rays, the *tasyīr* of the stars and the division of the houses.

You must know that, on the projection of rays, there are many methods. The best method and the most correct of what is said [on this] is what was mentioned by Ptolemy and Hermes. This is what Abū Maʿshar transmitted on the procedure for the rays taken from Ptolemy, it is the method used by the people nowadays. Abū Maʿshar said: if you want to make the rays of any star you wish from the ascensions, establish the ascendent and the tenth [house], and find the equivalent to the degree of the star in equal equatorial degrees (*daraj al-sawā' min al-falak al-mustaqīm*). Take what corresponds to it in ascensions and add [or subtract] to this, 60 for the sextile, 90 for the quartile, 120 for the trine; look for what corresponds to your sum [or subtraction] in right ascensions and transform it into ecliptical degrees. The result is the ray of the star using right ascensions, keep your final [ecliptical] degree. Then, do the same [as before] using oblique ascensions: if you found the ray of the star using right ascensions and using [also] oblique ascensions, and both rays agree within a single minute, this is the *equalized ray* of the star. [429] If the two [rays] diverge, subtract the smaller one from the larger one in ecliptical degrees [taken from the position] of the star and call this the *difference* between the two rays. Then, find the hours of *distance* from the star to the degree of the tenth [house], if it is above the earth, or its distance in hours to the degree of the fourth [house] if it is below, using right ascensions. This means that you take the ascensions of the degree of the tenth [house] and the ascensions of the degree of the star, if it is above the earth, and you do the same with the ascensions of the fourth [house] and the ascensions of the star, if it is below. What you obtain is the *distance* of the star to the cardine (*al-watad*) If the star is in the south-eastern quadrant [of the sphere] or in its nadir, which is the north-western quadrant, divide the *distance* of the star to the cardine by the length of a seasonal diurnal hour of the degree of the star (*azmān darajat nahār al-kawkab*). If the star is in the south-western quadrant or in its nadir, which is the north-eastern quadrant, divide the *distance* to the cusp by the length of a seasonal nocturnal hour of the degree of the star (*azmān darajat laylat al-kawkab*). The result in hours and their parts, if there is any fraction, is the hours of the *distance*. Then, divide the first *difference* by six parts and multiply the result by [430] the hours of the *distance* to the cardine; what results is the *equation*. Next, look at the ray of the star that you found using right ascensions. If it is smaller in degrees and closer to the position of the star than the ray that you found using oblique ascensions, add the *equation* to the ray that you found using right ascensions. If the ray that you found using right ascensions is larger in degrees and further from the position of the star than the ray that you found using oblique ascensions, subtract the *equation* from the ray that you found using right ascensions. The result, after its addition or subtraction, is the *equalized ray*. This is the method of the wise people in our days. In this procedure, there is some approximation if the star is in the two eastern quadrants, and a great flaw if the star is in the two western quadrants. The error happened to those who came before [us], among the Ancients, with the rays just because they found the rays following Ptolemy and Hermes using right and oblique ascensions, established their system on these two ascensions, and neglected the two western quadrants, they did not see that any ecliptical sign that rises with a known number [of ascensions] sets with the ascensions of its nadir. For this reason, they were entirely wrong concerning the rays when the star is in the two western quadrants. This [431] is because the resulting degrees of the *equation* are ecliptical degrees (*mustawiya*) and what is required for correctness is that the resulting degrees of the *difference* between the two rays be radial ecliptical degrees (*šuᶜāᶜiyya mustawiya*)[57].

We have made tables for the projection [...] of the rays for the latitude of Fes, compare [it] with that. [This is] the shape of my work: I put in the first [column of the table] the ecliptical degrees of the sign in which the star is, in its second [column] the ascensions of the midheaven, in its eighth [column] the oblique ascensions, in its ninth the length of the seasonal hours of the place. Following, I intended [to work with] the [first] degree of Aries, I took its corresponding right and oblique

[57] This last expression seems to refer to the use of *radial ascensions* for obtaining positions on the ecliptic. In the sense that Ibn Muᶜādh al-Jayyānī uses the expression, these *radial ascensions* are projections onto the equator of ecliptical points using incident horizons or aproximations to these using hour lines. What the text calls radial ecliptical degrees may be points on the ecliptic obtained by using the same procedure inversely. Cf. Casulleras, "Ibn Muᶜādh", 391-392.

ascensions […] and subtracted the smaller of it from the larger, being the remainder on the equator, I divided it in six parts, which are the hours of distance between [each one of] the two horizons[58] and the midheaven. I called the result *equation of the difference of the distance of one seasonal hour* from the midheaven. Then, I wrote down the ascensions of one degree of Aries in two places, subtracted the *equation of the difference of the distance of one seasonal hour* from one of them, and added it to the other, the remainder of the subtraction being the ascensions of the distance of one seasonal hour from the midheaven towards the eastern horizon [432], and the total of the addition the ascensions of the distance of one seasonal hour towards the western horizon. I wrote down each one of them [i.e. of the two results] in the table for the distance of the first hour from the midheaven on the side that I found suitable. Next, I multiplied the *equation of the distance of the hour* by two and subtracted it from the right ascensions in one of the two places and added it in the other, the remainder of the subtraction being the ascensions of the distance of two seasonal hours from the midheaven towards the eastern horizon, and the total of the addition the ascensions of the distance of two seasonal hours from the midheaven towards the western horizon. I did the same until completing the distance of six hours from the midheaven towards the eastern side and six hours towards the western side. This was the procedure: the ascensions of the distance of the sixth hour from the midheaven to the eastern side are the [oblique] ascensions of the place, and the ascensions of the distance of the sixth hour to the western side are the [oblique] descensions of the place, which correspond to the nadirs of the ascendent degree. This way, I put the ascensions corresponding to one degree and to two degrees of the sign of Aries, [and so on] until the end of the sign. If the oblique ascensions had been larger than the right ascensions[59] we would have divided this difference between them in six parts, then, we would have put [433] the right ascensions in two places and added one of the six parts to one of the two places and subtracted from the other, the total of the addition would have been the ascensions of the distance of one seasonal hour from the midheaven towards the

[58] That is, the eastern part of the horizon and its western part.

[59] This happens when the sign of $\delta(\lambda)$ is negative, that is, for $\lambda > 180°$.

eastern side, and the remainder of the subtraction the ascensions of the distance of one seasonal hour towards the western side. When I did this, [I considered] the stars projecting rays (*al-kawākib al-shu*ᶜ*ā*ᶜ*iyya*) on both sides from the midheaven and I knew that every hour that has its distance from the midheaven towards the western side is equivalent to another hour whose distance from the midheaven is towards the eastern side, that the degrees of the descensions of the western hour are equivalent to the degrees of the ascensions of the eastern hour, and that one of the two hours takes the place of the other like the ascendent and its nadir, [the ascendent] rising with the descensions that its nadir has in the western quadrant. [Consequently] I [only] wrote down the ascensions of the six eastern hours, for each one of the western hours corresponds to one of the eastern hours, since its distance is the same, like the western sixth [hour corresponds] to the eastern sixth [hour], the western fifth to the eastern fifth, [434] the fourth to the fourth, the third to the third, the second to the second and the western first to the eastern first, the midheaven is right ascensions, there is no deviation in it.

We made these tables for projecting the rays at the latitude of Fes. If you want to work with these tables and the degree of the star is in one of the two eastern quadrants, whose center is the center of the ascendent, you must know the hours of the distance of the star from the ascendent. To know this you take the ascensions of the ascendent degree and the ascensions of the degree of the star, [both taken] as right ascensions, and subtract the smaller from the larger. The remainder is the distance of the degree of the star from the ascendent degree. You divide this by the length of one diurnal seasonal hour of the degree of the star, if it is above the horizon, and by the length of one nocturnal seasonal hour of the star, if it is below the horizon. The quotient that you obtain will be the hours of distance from the eastern horizon. You do the same with the ascensions of the degree of the descendent and the ascensions of the degree of the star, if it is in the two western quadrants, the distance of the star from the degree of the descendent is divided by the length of one diurnal seasonal hour [435] of the star, if it is above the horizon, and by its nocturnal seasonal hour if it is below the horizon. The quotient will be the hours of distance from the descendent. When you know the hours of the distance of the star, on whichever side it is, using these procedures, look for the ecliptical degree that corresponds to the degree of the star in the sign in which it is on the page that has as a header "ascensions of ..." the sign of the star. Take what corresponds to ascensions in the table of the hours of the distance of the star from the eastern horizon and apply to it [that is, add to it or subtract to it, depending on whether you want a right or a left aspect] for the sextile 60, for the quartile 90, and for the trine 120, then look for what corresponds to the total amount on the next pages, in the sequence of the signs, in the column[60] of the hours of the distance of the star from the eastern horizon, where you reach the [entry of the table] corresponding to the number you have, transform it into ecliptical degrees [using the table inversely]. The result is the ray of that star. If the star is in the two western quadrants, whose center is the western horizon, look in the ecliptical degrees for what corresponds to the degree of the star in the sign it is on the page that has as a header "descensions of ..." the sign of the star, and determine its distance from the western horizon [and go on with the procedure] as explained above. Where your computation finishes, transform it into ecliptical degrees. The result is the ray of that star.

If there are any fractions with the hours and you want to adjust them, enter in another complete hour and take the difference between the two [hours] and determine the ratio of the fractions of the hour to its whole. If the ray of the first complete hour is smaller than the ray of the added [hour], add the adjustment of the fractions to the ray of the complete hour. If it is larger, subtract it. The result will be the equalized ray.

Most people in our days believe that these rays are the places of the lights of the stars in its own shape but I believe that the star only projects its rays for the sextile at [a distance of] 60°, for the quartile at 90°, and for the trine at 120° in ecliptical degrees, and the rays of these stars are only in ecliptical degrees. This is my method and my conviction, and the fruit of these tables is in the *tasyīr* of the stars among themselves and in [determinig] the value of the [arc of] *tasyīr* of one specific degree towards another degree which is known by the *tasyīr*, God, the Almighty, willing. God gives success to what is right[61].

[60] *Saṭr*, not in the sense of "line" but meaning "column" or "series of numbers".

[61] This same expression is used by Ibn ʿAzzūz at the end of his introduction to the *Zīj*. Cf. J. Samsó, "Ibn ʿAzzūz", 78, n.18.

Appendix 2. Edition of the Arabic text

[428] فصل في معرفة مطارح الأشعّة وتسيير الكواكب وتسوية البيوت.

اعلم أنّ في مطارح الأشعّة مذاهب كثيرة وأفضل المذاهب وأصحّ الأقاويل ما ذكره بطليموس وهرمس. وهذا ما نقله أبو معشر في عمل الشعاعات عن بطليموس به عمل أهل زماننا. قال أبو معشر إذا أردت عمل شعاع أيّ كوكب شئت بالمطالع فاقم الطالع والعاشر واطلب مثل درجة الكوكب في درج السواء من الفلك المستقيم وخذ ما بحيالها من المطالع وزد عليها للتسديس ص وللتربيع ض وللتثليث قك واطلب مثل ما يجتمع لك في مطالع الفلك المستقيم وحوّل ذلك إلى درج السواء فما خرج فهو شعاع الكوكب بمطالع الفلك المستقيم والدرج التي انتهيت اليها[62] واحفظه ثمّ افعل مثل ذلك بمطالع البلد فإذا عرفت شعاع الكوكب بمطالع الفلك المستقيم وبمطالع البلد واتّفق الشعاعان في دقيقة واحدة فذلك هو شعاع الكوكب المعدّل [429] وإن اختلفا فانقص الأقلّ من الأكثر في درج السواء للكوكب واعرف فضل ما بينهما وسمّه اختلاف ما بين الشعاعاين ثمّ اعرف ساعات[63] بعد الكوكب عن درجة العاشر إن كان فوق الأرض أو بعد ساعاته[64] عن درجة الرابع إن كان تحت الأرض بمطالع الفلك المستقيم وذلك بأن تأخذ مطالع درجة العاشر

[62] الذي انتهيت اليه MS التي انتهيت اليها

[63] شعاعات MS ساعات

[64] شعاعاته MS ساعاته

ومطالع درجة الكوكب إن كان فوق الأرض وكذلك تفعل بمطالع الرابع ومطالع الكوكب إن كان تحت الأرض فما خرج لك فهو بعد الكوكب عن الوتد فإن كان الكوكب في الربع الشرقيّ الجنوبيّ أو في نظيره الذي هو الربع الغربيّ الشماليّ فاقسم بعد الكوكب عن الوتد على أزمان <درجة> نهار الكوكب وإن كان الكوكب في الربع الغربيّ الجنوبيّ أو في نظيره الذي هو الربع الشرقيّ الشماليّ فاقسم البعد عن الوتد على أزمان درجة ليلة الكوكب فما خرج من الساعات وكسورها إن كان ثَمّ كسر فهي ساعات البعد. ثمّ اقسم الاختلاف الأوّل على ستّة أجزاء فما خرج اضربه في [430] ساعات البعد عن الوتد فما بلغ فهو التعديل. ثمّ انظر إلى شعاع الكوكب الذي عرفته بمطالع الفلك المستقيم فإن كان أقلّ درجا وأقرب إلى موضع الكوكب من الشعاع الذي عرفت بمطالع البلد فزد التعديل على الشعاع الذي عرفت بمطالع الفلك المستقيم وإن كان الشعاع[65] الذي عرفت بمطالع الفلك المستقيم أكثر درجا وأبعد عن موضع الكوكب من الشعاع الذي عرفت بمطالع البلد فانقص التعديل من[66] الشعاع الذي عرفت بمطالع الفلك المستقيم [وما بلغ اليه] بعد الزيادة عليه أو النقصان منه فهو الشعاع المعدّل. وهذا مذهب علماء أهل زماننا وفي هذا العمل تقريب يسير إذا كان الكوكب في الربعين الشرقيّين وخطأ كثير إذا كان الكوكب في الربعين الغربيّين وإنما وقع الغلط لمن تقدّم من الأوائل في الشعاعات لأنّهم وجدوا الشعاعات عن

[65] التعديل MS الشعاع

[66] I omit شعاع الفلك المستقيم بمطالع because this seems to be an error of the copyist.

بطليموس وهرمس بمطالع البلد والفلك المستقيم فأقاموا علمهم على هذين المطلعين وأغفلوا الربعين الغربيّين ولم ينظروا أنّ كلّ برج يطلع بعدد معلوم إنّه يغرب بمطالع نظيره فلذلك لا يصحّ لهم شيء من الشعاعات إذا كان الكوكب في الربعين الغربيّين. وذلك [431] أنّ درج التعديل الذي يخرج هي درج مستوية والواجب على الحقيقة أن يكون الدرج الخارج من اختلاف الشعاعين درجات شعاعيّة مستوية.

وقد عملنا جداول لمطارح [...] الشعاعات لعرض فاس فقس على ذلك. وصورة عملنا جعلت في الأوّل درج السواء للبرج الذي يكون فيه الكوكب وفي الثاني منه مطالع وسط السماء وفي الثامن منه مطالع البلد[67] وفي التاسع منه أزمان ساعات البلد ثمّ قصدت الدرجة من الحمل وأخذت ما بحيالها من <مطالع> الفلك المستقيم ومطالع البلد [...] ونقصت الأقلّ من الأكثر فكان الفضل للفلك المستقيم فقسمته على ستّة أجزاء التي هي ساعات بعد ما بين الأفقين ووسط السماء. وسمّيت الخارج تعديل اختلاف بعد ساعة زمانيّة عن وسط السماء. ثمّ أنزلت مطالع درجة من الحمل في ثلاثين ونقصت تعديل بعد ساعة زمانيّة من أحدهما وزدته على الثاني فكان المنقوص منه مطالع بعد ساعة[68] زمانيّة عن وسط السماء إلى الأفق الشرقيّ، [432] وكان المزاد عليه مطالع بعد ساعة زمانيّة إلى الأفق الغربيّ وأنزلت كلّ واحد منهما في جدول بعد الساعة الأولى عن وسط السماء في الناحية التي وجدتها

[67] البلد MS الفلك المستقيم, I follow the table, which contains oblique ascensions in the eighth column, for this correction.

[68] ساعة MS ساعة ثانية with ثانية crossed out.

له ثمّ ضربت تعديل بعد الساعة في اثنين ونقصت ذلك من مطالع الفلك المستقيم في أحد الموضعين وزدته على الآخر فكان المنقوص منه مطالع بعد ساعتين عن وسط السماء إلى الأفق الشرقيّ وكان المزاد عليه مطالع بعد ساعتين عن وسط السماء إلى الأفق الغربيّ وكذلك فعلت حتّى أكملت على بعد ستّ ساعات عن وسط السماء منه إلى ناحية المشرق وستّ[69] إلى ناحية المغرب. فكان هذا العمل: مطالع بعد الساعة السادسة عن وسط السماء إلى ناحية المشرق مطالع البلد[70] ومطالع بعد الساعة السادسة إلى ناحية المغرب مغارب البلد اللواتي عن نظير[71] الدرجة الطالعة وكذلك جعلت مطالع ما بحيال درجة ودرجتين من برج الحمل إلى آخر البرج. ولو كان مطالع البلد أكثر من مطالع الفلك المستقيم، لقسمنا هذا الفضل بينهما على ستّة أجزاء ثمّ جعلنا [433] مطالع الفلك المستقيم في موضعين وزدنا أحد الأقسام الستّة على أحد الموضعين ونقصناه من الثاني فكان يكون المزاد عليه مطالع بعد ساعة زمانيّة عن وسط السماء إلى ناحية المشرق، والمنقوص منه مطالع بعد ساعة زمانية إلى ناحية المغرب ولمّا عملت هذه الكواكب الشعاعيّة في أيّ الجهتين من وسط السماء وعلمت أنّ كلّ ساعة يكون بعدها عن وسط السماء إلى ناحية المغرب مساوية لساعة أخرى بعدها عن وسط السماء إلى ناحية المشرق وأنّ درج مغارب الساعة الغربيّة مساوية

[69] ستّة MS ستّ

[70] للبد MS البلد

[71] نظائر MS نظير

لدرج مطالع الساعة الشرقيّة وأنّ إحدى الساعتين تقوم مقام الأخرى كالطالع ونظيره الذي يطلع بمغارب نظيره القت الربع الغربيّ. وأنزلت مطالع الستّ ساعات الشرقيّة، إذ كلّ ساعة من الساعات الغربيّة مطابقة لساعة من الساعات الشرقيّة إذ كان بعدها بعدا واحدا كالسادسة الغربيّة للسادسة الشرقيّة والخامسة الغربيّة للخامسة الشرقيّة [434] والرابعة للرابعة والثالثة للثالثة والثانية للثانية والأولى الغربيّة للأولى الشرقيّة ووسط السماء هو مطالع الفلك المستقيم لا اختلاف[72] فيه.

وقد وضعنا هذه الجداول لمطارح الشعاعات على عرض فاس. فإذا أردت العمل بهذه الجداول وكانت درجة الكوكب في أحد الربعين الشرقيّين الذي مركزها مركز الطالع فاعلم ساعات بعد الكوكب من الطالع. ومعرفة ذلك أن تأخذ مطالع الجزء الطالع ومطالع جزء الكوكب في الفلك المستقيم وتنقص الأقلّ من الأكثر فما بقي فهو بعد الكوكب عن الجزء الطالع فتقسمه على أزمان درجة نهار الكوكب إن كان فوق الأفق وعلى أزمان درجة ليلة الكوكب إن كان تحت الأفق فما خرج لك من القسمة فهو ساعات بعد الكوكب عن الأفق الشرقيّ. وكذلك تفعل بمطالع جزء الغارب ومطالع جزء الكوكب إذا كان في الربعين الغربيّين ويقسم بعد الكوكب عن درجة الغارب[73] على أزمان درجة [435] نهار الكوكب إن كان فوق الأفق وعلى أزمان درجة ليله إن كان تحت الأفق فما

[72] لاختلاف MS لا اختلاف

[73] الطالع MS الغارب

خرج فهي ساعات بعد الكوكب عن الغارب[74] فإذا علمت ساعات بعد الكوكب في أيّ جهة كان بهذه الأعمال فاطلب في أيّ درجة السواء[75] مثل درجة الكوكب من البرج الذي هو فيه في الصفح الموضوع على رأسه مطالع برج الكوكب وخذ ما بحياله من المطالع في جدول ساعات بعد الكوكب عن الأفق الشرقيّ واحمل عليه للتسديس ص̅ وللتربيع ض̅ وللتثليث قك̅ ثمّ اطلب مثل الذي يجتمع في الصفحات التي تليها بما توالي البروج في سطر ساعات بعد الكوكب عن الأفق الشرقيّ فحيثما أصبت مثل العدد الذي معك فقوّسها الى درج السواء فما كان فهو شعاع ذلك الكوكب وإن كان الكوكب في الربعين الغربيّين اللّذيْن مركزهما الأفق الغربيّ، فاطلب في درج السواء مثل درجة الكوكب من البرج الذي هو فيه في الصفح الموضوع على رأسه[76] مغارب برج الكوكب واعرف بعده عن الأفق الغربيّ كما تقدّم فحيث انتهى بك العدد فقوّسه إلى درج السواء فما كان فهو شعاع ذلك الكوكب.

فإن كان مع الساعات كسر وأردت تعديلها فادخل بساعة أخرى تامّة وخذ فضل ما بينهما واعرف نسبة كسر الساعة من جملتها فإن كان شعاع الساعة التامّة الأولى أقلّ من شعاع الزائدة فزد تعديل الكسر على شعاع الساعة التامّة وإن كان أكثر انقصه منها فما بلغ فهو الشعاع المعدّل.

74 الغالب MS الغارب

75 I omit من الفلك المستقيم

76 الستّة MS رأسه

وأكثر أهل زماننا يعتقدون في هذه الشعاعات أنّها مواضع أنوار الكواكب في أشكالها وأنا أعتقد أنّ الكوكب لا يطرح شعاعه للتسديس إلّا ص̅ وللتربيع إلّا ض̅ وللتثليث إلّا قك̅ بدرج السواء وإنّما شعاعات هذه الكواكب بدرج السواء. هذا مذهبي واعتقادي وثمرة هذه الجداول في تسيير الكواكب بعضها إلى بعض وفي كم تسيير درجة معلومة إلى درجة أخرى معلومة بالتسيير إن شاء اللّه تعالى واللّه الموفّق للصواب.

Appendix 3. Edition and recomputation of the tables

Table of the ascensions of Aries and the descensions of Libra
in hours of distance with respect to the eastern horizon
(جدول مطالع الحمل ومغارب الميزان في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.[1]	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
1	0;55		0;52		0;49		0;46		0;44	-1	0;41	-1	0;39		15; 3	
2	1;50		1;45		1;29	+11	1;34	+1	1;28	+2	1;23	+2	1;18		15; 6	-1
3	2;45		2;37		2;29		2;21		2;13		2;46	-41	1;37	+20	15; 8	
4	3;40		3;29		3;19	-1	3; 8	-1	2;57	-1	2;46	-1	2;36		15;11	
5	4;35		4;22		4; 8	+1	3;55	+1	3;42	+1	3;28	+2	3;15		15;13	
6	5;30		5;14		4;58		4;42		4;26		4;10		3;54		15;16	
7	6;25		6; 6	+1	5;48	+1	5;29	+2	5;11	+2	4;52	+3	4;34		15;18	+1
8	7;20		6;58	+1	6;37	+1	6;16	+1	5;55	+1	5;53	-18	5;12	+1	15;21	
9	8;16		7;52		7;27	+1	7; 3	+1	6;39	+1	6;14	+2	5;51	+1	15;24	
10	9;11		8;44		8;17		7;50		7;23		6;56		6;30	+2	15;27	
11	10; 5	+1	8;34	+63	9; 6	+2	8;37	+2	8; 8	+2	7;38	+3	7; 9	+2	15;30	-1
12	11; 0	+2	10;28	+2	9;58		9;25	+1	8;53	+1	8;22		7;49	+2	15;32	
13	11;56	+1	11;21	+1	10;47		10;12		9;38	-1	9; 4	-2	8;29	+1	15;35	-1
14	12;51	+1	12;12	+3	11;37	+1	11; 1		10;23	+1	9;46	+1	9; 9	+1	15;37	
15	13;47	+1	13; 7	+1	12;27	+1	11;47	+	11; 8		10;28		9;49	+1	15;40	
16	14;42	+2	14; 0	+2	13;17	+3	12;35	+	11;53	+3	11;11	+3	10;29	+1	15;42	
17	15;39		14;59	-5	14; 9		13;24		12;39		11;54		11; 9	+1	15;45	
18	16;35		15;47		14;59		14;11		13;20	+3	12;36	-1	11;49	+1	15;48	
19	17;31		16;40	+1	15;51		15; 0	+	14;10	+1	13;20	+1	12;30		15;51	-1
20	18;27		17;34		16;41		15;48		14;55		14; 2		13;10	+1	15;53	
21	19;23		18;28		17;32	+1	16;56	-1	15;41	+2	14;46	+2	13;51		15;55	
22	20;19		19;21		18;23		17;25		16;27		15;29		14;32		15;58	
23	21;15	+1	20;14	+1	19;14		18;13		17;13	-1	16;12	-1	15;12	+1	16; 0	
24	22;12		21; 9		20; 6		19; 3		18; 0		16;57		15;54		16; 3	
25	23; 8	+1	22; 2	+1	20;57		19;51		18;46	-1	17;40	-1	16;35	+1	16; 6	
26	24; 5		22;57		21;48	+1	20;40	+	19;32	+1	18;24	+1	17;16	+1	16; 8	
27	25; 2		23;52		22;41	+1	21;30	+	20;19	+3	19; 8	+4	17;58	+1	16;11	
28	25;59		24;45	+1	23;32	+1	22;19	+	21; 2	+5	19;53	+1	18;40	+1	16;13	
29	26;26	+30	25;55	-15	24;24		23; 8		21;52		20;37	-1	19;22	+1	16;16	
30	27;53		26;34	+1	25;16	+1	23;58	+	22;40	+1	21;22	+1	20; 4	+1	16;18	

[1] For the recomputation and comparison of the tables I use the computer program Table Analysis (TA) developed by B. van Dalen at the Institute for the History of Science (Frankfurt a. M.). I give manuscript values and differences with the recomputation.

Ascensions of Taurus and descensions of Scorpius in hours of distance with respect to the eastern horizon

(مطالع الثور ومغارب العقرب في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
31	28;50	+1	27;29	+1	26; 9		24;48		23;28	-1	22; 7	-1	20;47	+1	16;20	+1
32	29;48		28;25		27; 2		25;39		24;16		22;53		21;30	+1	16;23	
33	30;46		29;20	+1	27;35	+21	26;29	+	25; 4	+2	23;38	+3	22;13	+1	16;26	-1
34	31;43	+1	30;15	+1	28;47	+1	27;19	+	25;51	+1	24;33	-9	22;56	+1	16;28	
35	32;41	+1	31;11	+1	29;40	+2	28;10	+	26;40	+2	25; 9	+3	23;39	+1	16;30	
36	33;40		32; 8	-1	30;35	-1	29; 2	-	27;30	-2	25;57	-2	24;25	-1	16;33	
37	34;38		33; 3		31;28		29;53		28;18		26;43		25; 8		16;35	
38	35;36	+1	33;59	+1	32;22	+1	30;44	+	29; 7	+2	27;29	+3	25;55	-2	16;38	-1
39	36;35		34;55		33;15		31;35		29;56	-1	28;16	-1	26;37		16;40	
40	37;34		35;52		34;10		32;28		30;46		29; 4		27;22		16;42	
41	38;33		36;49		35; 5		33;21		31;30	+7	29;52	+1	28; 7	+1	16;45	-1
42	39;32		37;45		35;58		34;11		32;25	-1	30;18	+19	28;52	+1	16;47	-1
43	40;31	+1	38;42	+1	36;54		35; 4	+	33;15	+1	31;26	+1	29;37	+2	16;49	
44	41;30	+1	39;39	+1	37;48	+1	35;57	+	34; 6	+1	32;15	+1	30;24	+1	16;51	
45	42;30	+1	40;37	+1	38;44	+1	36;50	+	34;48	+11	33; 5	+1	31;12		16;53	
46	43;30	+1	41;35	+1	39;40	+1	37;45	+	35;50	+1	33;55	+1	32; 0	-1	16;55	
47	44;30	+1	42;32	+1	40;35		38;37		36;40	-1	34;52	-11	32;45	+1	16;58	-1
48	45;30	+1	43;30	+1	41;30	+1	39;30	+	37;30	+1	35;31		33;32	+2	17; 0	
49	46;31		44;29		42;27		40;25		38;24	-1	36;22	-1	34;21	+1	17; 2	
50	47;31	+1	45;27	+1	43;24		41;19	+	39;15	+1	37;11	+1	35; 8	+2	17; 4	
51	48;32	+1	46;26	+1	44;20	+1	42;14	+	40; 8	+1	38; 2	+1	35;57	+2	17; 6	
52	49;34		47;25	+1	45;17	+1	43;10		41; 2		38;54		36;47	+1	17; 8	
53	50;34	+1	48;24	+1	46;15		44; 4	+	41;55		39;45		37;36	+1	17;10	
54	51;36		49;25		47;13	+1	45; 1	+	42;49	+3	40;37	+4	38;26	+1	17;12	
55	52;37	+1	50;23	+1	48;10		45;56		43;43	-1	41;29	-1	39;16	+1	17;14	-1
56	53;39		51;24		49; 9		46;53	+	44;37	+2	42;27	-3	40; 7	+1	17;15	
57	54;41		52;23	+1	50; 6	+1	47;49	+	45;32	+1	43;15	+1	40;58	+1	17;17	
58	55;43		53;24		51; 5		48;46		45;27	+60	44; 8		41;50		17;19	
59	56;46	-1	54;24		52; 3		49;43	-	47;22	-1	45; 1	-1	42;41	+1	17;21	
60	57;47	+1	55;24	+2	53; 2	+2	50;40	+	48;17	+3	45;55	+3	43;33	+2	17;22	

Ascensions of Gemini and descensions of Sagittarius
in hours of distance with respect to the eastern horizon
(مطالع الجوزاء ومغارب القوس في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
61	58;50		56;26		54; 2		51;38		49;14		46;50		44;26	+1	17;24	
62	59;53		57;28	-1	55; 3	-2	52;39	-	50;14	-5	47;48	-5	45;25	-5	17;26	-1
63	60;56		58;29		56; 2		53;36	-	51; 6	+2	48;46	-5	46;13	+1	17;27	
64	61;59		59;30	+1	57;42	-39	55; 3	-2	52; 9	-2	49;42	-3	47; 8		17;29	
65	63; 2		60;32		58; 2		55;32		53; 2		50;33	-1	48; 2	+1	17;30	
66	64; 5	+1	61;33	+2	59; 2	+2	56;30	+	53;59	+3	51;27	+4	48;56	+1	17;32	-1
67	65;13	-4	62;36		60; 3		57;30		54;57		52;25	-1	49;52	+1	17;33	
68	66;13		63;38	+1	61; 4	+1	58;30	+	55;56	+1	53;22	+1	50;48	+1	17;35	-1
69	67;18	-1	64;41	+1	62; 6	+1	59;30	+	56;55	+2	54;19	+3	51;46	-1	17;36	-1
70	68;21		65;44		63; 7		60;31	-	57;54	-1	55;17	-1	52;42	-1	17;37	
71	69;25		66;47		64; 9		61;31		58;58	-5	56;14	+1	53;38	+1	17;38	
72	70;29		67;50		65;11		62;32		59;53		57;14		54;35	+1	17;39	
73	71;33		68;53		66;13		63;33		60;53		58;13		55;33	+1	17;40	
74	72;38		69;56	+1	67;15	+1	64;34	+	61;53	+1	59;12	+1	56;35	-2	17;41	
75	73;42		71; 0		68;18		65;36		62;52	+2	60;12		57;30	+2	17;42	
76	74;47		72; 6	-2	69;22	-1	66;39	-	63;56	-1	61;13	-1	58;30	+1	17;43	
77	75;52		73; 8		70;24		67;41	-	64;57	-1	62;14	-2	59;30	+1	17;44	
78	76;57		74;12	+1	71;28	+1	68;43	+	65;59	+2	63;14	+3	60;30	+1	17;45	-1
79	78; 2		75;16	+1	72;32		69;46	+	67; 1	+1	64;16	+1	61;31	+1	17;45	
80	79; 7		76;21		73;36	-1	70;50	-	68; 4	-1	65;18	-1	62;30	+3	17;46	
81	80;12		77;25	+1	74;39	+1	71;52	+	69; 6	+2	66;19	+3	63;33	+1	17;47	-1
82	81;17		78;30		75;43		72;56		70; 9		67;22		64;35	+1	17;47	
83	82;22		79;34	+1	76;47	+1	73;59	+	71;15	-1	68;24	+3	65;37	+2	17;48	-1
84	83;28		80;40		77;52		75; 4		72;16		69;25	+3	66;40	+2	17;48	
85	84;32	+1	81;44	+1	78;56	+1	76; 8	+	73;19	+2	70;31	+2	67;48	-3	17;48	
86	85;38		82;49	+1	80; 1	+1	77;12	+	74;21	+5	71;35	+3	68;47	+2	17;48	
87	86;43	+1	83;54	+1	81; 6		78;17		75;28		72;39		69;51	+2	17;49	-1
88	87;49		85; 1	-1	82;12	-1	79;23	-	76;34	-1	73;45	-1	70;56	+1	17;49	
89	88;54	+1	86; 5	+1	83;16	+1	80;28		77;38	+1	74;49	+1	72; 0	+2	17;49	
90	90; 0		87;11		84;22		80;33	+6	78;44		75;55		73; 6	+1	17;49	

Ascensions of Cancer and descensions of Capricorn
in hours of distance with respect to the eastern horizon
(مطالع السرطان ومغارب الجدي في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
91	91; 5		88;14	+2	85;27		82;39	-	79;50	-1	77; 1	-1	74;12	+1	17;49	
92	92;11		89;22		86;34	-1	83;45	-	80;56	-1	78; 7	-1	75;18	+1	17;49	
93	93;16		90;27		87;39	-1	84;50	-	82; 2	-2	79;13	-2	76;22	+3	17;49	-1
94	95;22	-60	91;34		88;46		85;57	+	83; 9	+1	80;20	+2	77;32		17;48	
95	95;17	+10	92;39		89;51		87; 3		84;14	+1	81;27		78;39		17;48	
96	96;32		93;44		90;56		88; 3	+	85;21	-1	82;33	-1	79;46	+1	17;48	
97	97;38		94;50	+1	92; 6	-2	89;15	+	86;28	+2	83;40	+3	80;53	+1	17;48	-1
98	98;43		95;56		93; 9		90;22		87;35		84;48		82; 1	+1	17;47	
99	99;48		97; 1	+1	94;15	+1	91;28	+	88;42	+2	85;59	-1	83; 9	+2	17;47	-1
100	100;53		98; 8	-1	95;22	-1	92;39	-	89;50	-1	87; 4	-1	84;18	+1	17;46	
101	101;58		99;12	+1	96;27	+1	93;42	+	90;57	+1	88;12	+1	85;27	+1	17;46	-1
102	103;53	-50	100;18	+1	97;34	+1	94;49	+	92; 5	+2	89;20	+3	86;36	+2	17;45	-1
103	104;18	-10	101;24	+1	98;41	+1	95;56	+	93;13	+3	90;29	+4	87;47		17;44	
104	105;13		102;31	-1	99;48	-1	97; 5	-	94;22	-1	91;39	-1	88;56	+1	17;43	
105	106;18		103;36		100;54		98;12		95;30		92;48		90; 6	+1	17;42	
106	107;22		104;42	-1	102; 0		99;20	-	96;39	-1	93;58	-1	91;17		17;41	
107	108;27		105;47		103; 7		100;27		97;57	-10	95;50	-43	92;27		17;40	
108	109; 9	+22	106;42	+10	104;13		101;36	-	98;55		96;16		93;37	+1	17;39	
109	110;35		107;58	-1	105;20	-1	102;42	-	100; 4	-1	97;26	-1	94;48	+1	17;38	
110	111;31	+8	109; 8	-6	106;26	-1	103;49	-	101;12	-1	98;36	-2	95;59	+1	17;37	
111	112;53	-10	110; 8		107;32	+1	104;57	+	102;21	+2	99;46	+2	97;10	+1	17;35	
112	113;39	+8	111;13		108;39		106; 5		103;31		100;56	+1	98;22	+1	17;34	
113	114;53	-2	112;18		109;44	+1	107;13	-	104;40	-1	102; 7	-1	99;34		17;33	
114	115;54		113;22	+1	110;51	+1	108;20	+	105;49	+1	103;18	+1	100;46		17;32	-1
115	116;58		114;28		111;58		109;28		106;58		104;28		101;58		17;31	-1
116	118;54	-53	115;24	+8	113; 3		110;36	-	108; 7	-2	105;39	-3	103;10		17;29	
117	119;58	-54	116;37		114;11	-1	111;23	+2	109;16		106;49		104;22		17;27	
118	120; 7		117;42	-1	115;16	-1	112;51	-	110;22	+1	108; 0	-3	105;34		17;26	-1
119	121; 9	+1	118;48	-2	116;21	+1	113;57	+	111;23	+11	109; 9	+1	106;46	+1	17;25	-1
120	122;12		119;50		117;28		116; 6	-6	112;44		110;22		107;59		17;22	

Ascensions of Leo and descensions of Aquarius
in hours of distance with respect to the eastern horizon
(مطالع الأسد ومغارب الدلو في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
121	123;15		121;55	-61	118;34	-1	117;13	-6	114;53	-62	111;30		109;11		17;21	
122	124;17		121;59	-1	119;40	-1	117;21	-	115; 5	-4	112;43	-1	110;24		17;19	
123	125;19		123; 1	+1	120;44	+1	118;27	+	116;10	+1	113;53	+1	111;36	+1	17;17	
124	126;21		124; 5	+1	121;50	+1	119;35	+	117;20	+1	115; 5	+1	112;49		17;15	
125	127;23	-1	125; 9		122;56		120;46	-	118;29	+1	116;15	+2	114; 2		17;14	-1
126	128;24		126;12	+1	124; 1	+1	121;49	+	119;18	+22	117;26	+3	115;15		17;12	
127	129;26	-1	127;16	-1	125; 6	-1	122;56	-	120;47	-2	118;37	-2	116;28		17;10	
128	130;27	-1	128;19	-1	126;11	-1	124; 3	-	121;56	-2	119;48	-2	117;41		17; 8	
129	131;28	-1	129;22	-1	127;16	-1	125;10	-	123; 4	-1	120;58	-1	118;48	+5	17; 6	
130	132;29	-1	130;25	-1	128;21	-1	126;17	-	124;14	-2	122;10	-2	120;46	-40	17; 4	
131	133;29		131;27		129;26	-1	127;23		125;22	-1	123;20	-1	121;19		17; 2	
132	134;30	-1	132;30	-1	130;30	-1	128;30	-	126;31	-2	124;31	-2	122;32		17; 0	
133	135;30	-1	133;33	-1	131;36	-1	129;39	-	127;41		125;44		123;46	-1	16;58	-1
134	136;30	-1	134;35	-1	132;40	-1	130;45	-	128;50	-1	126;45	+9	126; 0	-62	16;58	-3
135	137;30	-1	135;37	-1	133;44	-1	131;51	-	129;58	-1	128; 5	-1	126;12	-2	16;58	-5
136	138;29		136;38		134;47		132;56		131; 5		129;14		127;24	-1	16;51	
137	139;28		137;40	-1	135;51	-1	134; 2	-	132;13	-1	130;24	-1	128;31	+5	16;49	
138	140;28		138;40	+2	136;53	+3	135; 6	+	133;19	+5	131;35	+3	129;45	+4	16;47	-1
139	141;27		139;42	+1	137;58	+1	136;14	+	134;29	+2	132;41	+6	131; 1	+1	16;45	-1
140	142;26		141;44	-60	139; 2		137;20		135;38		133;56		132;14		16;42	
141	143;25		142;45	-60	140; 5		138;25		136;46	-1	135; 6	-1	133;27		16;40	
142	144;23		143;47	-61	141;10	-1	139;33	-	137;55		136;18		134;40	-1	16;38	-1
143	145;22		144;47	-60	142;12		140;37		139; 2		137;27		135;52		16;35	
144	146;20		144;47		143;16	-2	141;43	-	140;10	-2	138;18	+17	137; 5	-1	16;33	
145	147;18		145;48		144;19	-1	142;49	-	141;18		139;48		138;17		16;30	
146	148;16		146;49	-1	145;22	-2	143;55	-	142;26	-2	140;58	-2	139;30	-1	16;28	
147	149;14		147;49		146;24		144;58	+	143;32	+2	142; 7	+2	140;41	+1	16;26	-1
148	150;12		148;49		147;25	+1	146; 2	+	144;39	+1	143;16	+1	141;54		16;23	
149	151; 9		149;49		148;29		147; 9		145;48	+1	144;28	+1	143; 7	-1	16;20	+1
150	152; 7		148;49	+120	149;31		148;12	+	146;54	+1	145;36	+1	144;18		16;18	

Ascensions of Virgo and descensions of Pisces in hours of distance with respect to the eastern horizon

(مطالع السنبلة ومغارب الحوت في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
151	153; 4		151;48		150;32		149;17	-	148; 1	-1	148;45	-121	145;30		16;16	
152	154; 1		152;48		151;36	-1	150;22		149; 9		148;55	-59	146;42		16; 3	+10
153	154;58		153;47		152;36		151;25		150;15	-1	149; 4	-1	147;54		16; 9	+2
154	155;55		154;45	+2	153;39		152;31		151;23		150;14	+1	149; 6		16; 8	
155	156;51		155;41	+4	154;41	-2	153;36	-	152;30	-3	151;20	+1	150;39	-21	16; 5	+1
156	157;48		156;45		155;42		154;39		153;36		152;33		151;30		16; 3	
157	158;44		157;45	-1	156;44		155;44		154;43	+1	153;42	+2	152;42		16; 1	-1
158	159;41		158;48	-5	157;45		156;47		155;50	-1	154;52	-1	153;34	+19	15;58	
159	160;37		159;42		158;47		157;41	+1	156;56	+1	156; 0	+2	155; 5		15;55	
160	161;33		160;41	-1	159;48	-1	158;55	-	158; 2	-1	157; 9	-1	156;16	+1	15;53	
161	162;29		161;39		160;48	+1	159;58	+	159; 8	+1	158;18	+1	157;28		15;50	
162	163;25		162;37		161;49		161; 2	-	160;14	-1	159;26	-1	158;29	+11	15;48	
163	164;21		163;36		162;51		162; 6		161;21		160;36		159;51		15;45	
164	165;16		164;34		163;51	+1	163; 9	+	162;27	+1	161;45	+1	161; 3	-1	15;42	
165	166;12		165;32		164;52		164;13	-	163;34	-2	162;54	-2	162;15	-1	15;40	
166	167; 8		166;31		165;54		165;17		164;40		164; 3		163;26	-1	15;37	
167	168; 3		167;29		166;55		166;20	+	165;46	+1	165;11	+2	164;37	-1	15;35	-1
168	168;58		168;27	-1	167;55	-1	167;23	-	166;52	-2	166;20	-2	165;48	-1	15;32	
169	169;54		169;25		168;56		168;27		168;58	-60	167;27	+2	166;59		15;30	-1
170	170;46	+3	170;53	-30	169;57		169;30	+	169; 3	+2	168;37	+2	168;10		15;27	
171	171;44		171;21	-1	170;46	+10	170;33	-	170; 9	-1	169;45	-1	169;21		15;22	+2
172	172;40		172;19		171;58		171;36	+	171;15	+1	170;54	+1	170;32		15;21	
173	173;35		173;16		172;57		172;39	-	172;20	-1	172; 1	-1	171;43		15;18	+1
174	174;30		174;14		173;58		173;42		173;24	+2	173;10		172;54		15;16	
175	175;24	+1	175;12		174;59		174;45	+	174;32	+1	174;18	+2	174;35	-30	15;13	
176	176;20		176; 9		175;58		175;48	-	175;37	-1	175;27	-2	175;16		15;11	
177	177;15		177; 7		176;59		176;51		176;43		176;35		176;27		15; 8	
178	178;10		178; 5		177; 0	+60	177;54	+	177;49	+1	177;43	+2	177;38		15; 6	-1
179	179; 5		179; 2		178;59		178;57	-	178;52	+1	178;52	-2	178;49		15; 3	
180	180; 0		180; 0		180; 0		180; 0		180; 0		180; 0		180; 0		15; 0	

Ascensions of Libra and descensions of Aries
in hours of distance with respect to the eastern horizon
(مطالع الميزان ومغارب الحمل في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
181	180;52	+3	180;56	+2	181; 0	+1	181; 4		181; 5	+2	180; 8	+62	181;11		14;55	+2
182	181;50		181;55		182; 1	-1	182; 6	-	182;15	-5	182;17	-2	182;52	-30	14;54	+1
183	182;45		182;53		183; 1		183; 9		183;17		183; 9	+16	183;33		14;52	
184	183;40		183;51		184; 1	+1	184;12	+	184;22	+2	184;33	+2	184;44		14;49	
185	184;35		184;48		185; 2	-1	185;15	-	185;29	-2	185;42	-2	185;55		14;47	
186	185;30		185;46		186; 2		186;18		186;34		186;50		187; 6		14;44	
187	186;25		186;44		187; 2	+1	187;21	+	187;39	+2	187;58	+2	188;16	+1	14;42	-1
188	187;20		187;41		188; 3	-1	188;24	-	188;46	-2	189; 7	-2	189;28		14;39	
189	188;15	+1	188;39	+1	189; 3	+1	189;27	+	189;51	+1	190;15	+1	190;49	-10	14;34	+2
190	189;11		189;37		190; 4	-1	190;30	-	190;57	-2	191;23	-2	191;50		14;33	
191	190; 6		190;35		191; 4		191;32	+	192; 3	-1	192;22	+9	193; 1		14;30	+1
192	191; 1	+1	191;33	+1	192; 4	+2	192;36	+	193; 8	+2	193;40	+2	194;12	+1	14;28	
193	191;57		192;30	+1	193; 5		193;39		194;14	-1	194;48	-1	195;23	+1	14;25	+1
194	193;52	-60	193;29		194; 6		194;43		195;20		195;57		196;54	-19	14;23	
195	194;44	-56	194;27	+1	195; 7	+1	195;46	+	196;26	+2	197; 5	+3	197;45	+1	14;20	
196	195;44	-60	195;25	+1	196; 8		196;50		197;33	-1	198;15	-1	198;57	+1	14;18	
197	196;39	-60	196;24		197; 9		197;54		198;29	+10	199;24		200; 9		14;15	
198	196;55	-20	197;23		198;10	+1	198;58	+	199;46	+1	200;34	+1	201;21	-1	14;12	
199	197;31		198;21		199;11		200; 1		200;52	-1	201;42	-1	202;32		14; 9	+1
200	198;27		199;20		200;13		201; 5	+	201;58	+1	202;51	+1	203;44	-1	14; 7	
201	199;23		200;18		201;14	-1	202; 9	-	203; 5	-2	204; 0	-2	204;55		14; 5	
202	200;19		201;17		202;15		203;12	+	204;10	+1	205; 8	+1	206; 6	+1	14; 2	
203	201;15	+1	202;16		203;16		204;17	-	205;17	-1	206;18	-2	207;18		13;59	+1
204	202;12		203;15		204;18		205;21		206;24		207;27		208;30		13;57	
205	203; 8	+1	204;14	+1	205;19	+2	206;25	+	207;30	+3	208;38	+1	209;45	-3	13;52	+2
206	204; 5		205;13		206;21		207;30	-	208;38	-1	209;46	-1	210;54		13;52	
207	205; 2		206;13		207;23	+1	208;54	-1	209;44	+2	210;59	-2	212; 6		13;59	-10
208	205;59		207;12		208;24	+1	209;39	-	210;52	-1	212; 5	-1	213;18		13;47	
209	206;56		208;12		209;28		210;43	+	211;59	+1	213;15	+1	214;30		13;44	
210	207;53		209;11		210;29		211;47		213; 6	-1	214;24	-1	215;42		13;44	-2

Ascensions of Scorpius and descensions of Taurus
in hours of distance with respect to the eastern horizon
(مطالع العقرب ومغارب الثور في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
211	208;51		210;51	-40	211;31		212;52	-	214;12	-1	215;24	+7	216;33	+21	13;39	
212	209;48		211; 4	+7	212;29	+5	213;57		215;58	-38	216;43		218; 6		13;36	+1
213	210;46		212;11		213;37	-1	215; 2	-	216;28	-2	217;53	-2	219;19	-1	13;34	+1
214	211;43	+1	213;11	+1	214;38	+2	216; 6	+	217;34	+2	219; 2	+2	220;30	+1	13;32	
215	212;41	+1	214;11	+1	215;42		217;12		218;43	-1	220;13	-1	221;43		13;30	
216	213;40		215;12	+1	216;45	+1	218;17	+	219;50	+2	221;22	+3	222;55	+1	13;27	
217	214;38		216;13		217;48		219;23		220;46	+12	222;33		224; 8		13;25	
218	215;36	+1	217;13	+1	218;51		220;28		222; 6	-1	223;43	-1	225;20	+1	13;22	+1
219	216;35		218;15		219;54	+1	221;34	+	223;13	+2	224;53	+2	226;33		13;20	
220	217;34		219;16		220;58		222;40		224;22		226; 4		227;46		13;18	
221	218;33		220;17		222; 2	-1	223;46	-	225;31	-2	227;15	-2	228;59	-1	13;16	
222	219;32		221;19	-1	223; 5	-1	224;52	-	226;38	-2	228;25	-3	230;12	-1	13;13	+1
223	220;31	+1	222;20	+1	224; 9	+1	225;57	+	227;46	+2	229;35	+2	231;24		13;11	
224	221;30	+1	223;21	+1	225;12	+1	227; 3	+	228;54	+1	230;45	+1	232;36	+1	13; 9	
225	222;30	+1	224;23	+1	226;16	+1	228; 9	+	230; 2	+1	231;55	+1	233;48	+2	13; 7	
226	223;30	+1	225;25	+1	227;20	+1	229;15	+	231;10	+1	233; 5	+1	235; 0	+2	13; 5	
227	224;30	+1	226;27	+1	228;25		230;22		232;20	-1	234;17	-1	236;14	+1	13; 2	+1
228	225;30	+1	227;30		229;29		231;29	-	233;28	-1	235;28	-2	237;28		13; 0	
229	226;31		228;33		230;35		232;36	+	234;37	+2	236;19	+22	238;41		12;58	
230	227;31	+1	229;35	+1	231;38	+2	233;42	+	236;46	-58	237;50	+2	239;54		12;56	
231	228;32	+1	230;38	+1	232;44	+1	234;49	+	237;55	-58	239; 1	+2	241; 7		12;54	
232	229;33	+1	231;41	+1	233;48	+2	235;56	+	238; 3	+3	240;12	+2	242;19		12;52	
233	230;34	+1	232;44	+1	234;53	+2	237; 3	+	239;12	+3	241;22	+3	243;32		12;50	
234	231;36		233;47		235;59	-1	238;10	-	240;22	-2	242;23	+8	244;45		12;48	
235	232;37	+1	234;50	+1	237; 4		239;17		241;31	-1	243;44	-1	245;58		12;47	
236	233;39		235;54		238;10	-1	240;22	+	242;41	-2	244;56	-2	247;11		12;45	
237	234;41		236;58		239;16	-1	241;33	-	243;50	-1	246; 7	-1	248;24	-1	12;43	
238	235;43		238; 2		240;21		242;40		244;58	+1	247;17	+1	249;36		12;41	
239	236;45		239; 5	+1	241;26	+1	243;46	+	246; 7	+2	248;28	+2	250;49		12;39	
240	237;47	+1	240; 9	+1	242;32		244;54		247;17	-1	249;39	-1	252; 1		12;38	

Ascensions of Sagittarius and descensions of Gemini in hours of distance with respect to the eastern horizon

(مطالع القوس ومغارب الجوزاء في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
241	238;50		241;14		243;38		246; 2		248;26		250;55	-5	253;14	-1	12;36	
242	239;53		242;18	+1	244;44	+1	247; 9	+	249;35	+2	252; 1	+2	254;25	+1	12;36	-1
243	240;56		243;23		245;50		248;18	-	250;45	-1	253;12	-1	255;39	-1	12;36	-3
244	241;59		244;28		246;56	+1	249;24	+	251;53	+2	254;22	+2	256;50		12;35	-4
245	243; 2		245;32		248; 2		250;32		253; 2		255;32		258; 2		12;33	-3
246	244; 5	+1	246;36	+1	249; 8		251;39		254;11	-1	256;42	-1	259;14		12;31	-2
247	245; 9		247;42		250;15		252;47	+	255;20	+1	257;53	+1	260;26		12;30	-3
248	246;13		248;47		251;21		253;56	-	256;30	-1	259; 6	-3	261;38	-1	12;28	-2
249	247;17		249;52		252;28	-1	255; 3	-	257;39	-2	260;15	-3	262;30	+19	12;27	-2
250	248;20	+1	250;56	+1	253;34	-1	256;10	-	258;47	-2	261;24	-3	264; 1	-1	12;26	-3
251	249;25		252; 3		254;40	+1	257;18	+	259;56	+1	262;34	+1	265;12	-1	12;24	-2
252	250;29		253;18	-10	255;47		258;26		261; 5		263;44		266;23	-1	12;23	-2
253	251;33		254;13		256;53		259;33		262;13		264;53		267;33		12;22	-2
254	252;37	+1	255;18	+1	257;59	+1	260;40	+	263;22		265; 2	+61	268;43		12;21	-2
255	253;42		256;29	-5	259; 6		261;48		264;30		267;12		269;54	-1	12;20	-2
256	254;47		257;30		260;13		262;55	+	265;38	+1	268;21	+1	271; 4	-1	12;19	-2
257	256;52	-60	258;36		261;21	-1	264; 3	+	266;46	+2	269;30	+2	272;14	-1	12;18	-2
258	259;57	-180	259;41		262;26	-1	265;10	-	267;55	-2	270;40	-3	273;24	-2	12;17	-1
259	260; 2	-120	260;46	+1	263;23	+9	266;18	-	269; 2		271;48	-1	274;33	-1	12;16	-1
260	261; 7	-120	261;53		264;38	+1	267;24	+	270;10	+1	272;56	+1	275;42	-1	12;15	-1
261	262;12	-120	262;58		265;45	-1	268;24	+	271;18	-2	274; 5	-3	276;51	-2	12;15	-1
262	263;17	-120	264; 4		266;51		269;28	+1	272;25		275;12		277;59	-1	12;14	-1
263	264;22	-120	265; 9		267;56		270;45	-	273;32	-2	276;20	-3	279; 7	-1	12;13	
264	265;27	-119	266;15	+1	269; 3	+1	271;50	+	274;38	+2	277;26	+2	280;14	-1	12;13	-1
265	266;32	-119	267;20	+1	270; 9		271;57	+6	275;45		278;33		281;21		12;12	
266	267;38	-120	268;26		271;15	-1	272; 4	+11	276;52	-2	279;40	-2	282;28		12;12	
267	268;43	-119	269;31	+2	272;20	+2	275; 8	+	277;57	+3	281;44	-55	283;55	-20	12;12	
268	269;49	-120	270;38		273;26	+1	276;15	+	279; 4	+1	281;53	+1	284;42	-1	12;12	-1
269	270;54	-119	271;44		274;32	+1	277;21	+	280;10	+1	282;59	+1	285;48	-1	12;11	
270	270; 0		272;49		275;38		278;27		281;16		284; 5		286;54	-1	12;11	

Ascensions of Capricorn and descensions of Cancer
in hours of distance with respect to the eastern horizon
(مطالع الجدي ومغارب السرطان في ساعات البعد عن الأفق الشرقي)

λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
271	271; 6	-1	273;55	-1	276;44	-1	279;33	-	282;22	-1	285;11	-1	288; 0	-2	12;11	
272	272;11		275; 0		277;48	+1	280;37	+	283;26	+1	286;15	+1	289; 4	-1	12;11	
273	273;17	-1	276; 6	-1	278;54		281;42	+	284;31	+1	287;20	+1	290; 9	-2	12;11	+1
274	274;22		277;11	-1	279;59	-1	282;48	-	285;35	-1	288;25	-3	291;13	-2	12;12	
275	275;28	-1	278;16	-1	281; 5	-2	283;53	-	286;41	-2	289;29	-2	292;17	-2	12;12	
276	276;33	-1	279;21	-1	282; 9	-1	284;56		287;44		290;32		293;20	-2	12;12	
277	277;38		280;25		283;13	-1	286; 0	-	288;48	-2	291;36	-3	294;23	-2	12;12	+1
278	278;28	+15	281;30		284;17		287; 4		289;51		292;38		295;25	-1	12;13	
279	279;48		282;35	-1	285;21	-1	288; 8	-	290;54	-2	293;40	-2	296;27	-1	12;13	+1
280	280;53		283;39		286;25		289;10	+	291;56	+1	294;42	+1	297;28	-1	12;14	
281	281;58		284;43		287;28		290;14	-	292;59	-1	295;44	-1	298;29	-1	12;15	
282	283; 3		285;48	-1	288;32	-1	291;17	-	294; 1	-2	296;46	-3	299;30	-1	12;15	+1
283	284; 8		286;52		289;35	+1	292;19	+	295; 2	+2	297;46	+2	300;30	-1	12;16	
284	285;13		287;56		290;39		293;21	+	296; 4	+1	298;46	+2	301;30	-1	12;17	
285	286;18		289; 0		291;42		294;24		297; 6		299;48		302;30	-2	12;18	
286	287;23	-1	290; 4	-1	292;45	-1	295;26	-	298; 7	-1	300;48	-1	303;29	-2	12;19	
287	288;27		291; 7		293;47		296;27		299; 7		301;47		304;27	-1	12;20	
288	289;31		292;10		294;49		297;27	+	300; 6	+1	302;45	+1	305;29	-5	12;21	
289	290;35		293;13		295;50	+1	298;28	+	301; 6	+1	303;45		306;22	-1	12;22	
290	291;39		294;16		296;52	+1	299;29	+	302; 5	+2	304;45	-1	307;19		12;23	
291	292;43		295;19	-1	297;54	-1	300;30	-	303; 5	-2	305;41	-3	308;16	-1	12;24	+1
292	293;47		296;21		298;59	-4	301;30	-	304; 5	-2	306;38	-1	309;12	-1	12;26	
293	294;51		297;23	+1	299;57		302;28	+	305; 0	+3	307;33	+3	310; 5	+2	12;27	
294	295;55	-1	298;26	-1	300;57	-1	303;28	-	306;55	-57	308;30	-1	311;28	-25	12;28	+1
295	296;58		299;28		301;58		304;28		306;58		309;29	-1	311;58	-1	12;30	
296	298; 1		300;29	+1	302;58	+1	305;26	+	307;55	+2	310;24	+2	312;52		12;31	
297	299; 4		301;31		303;59	-1	306;26	-	308;53	-1	311;20	-1	313;47	-1	12;33	
298	300; 7		302;32		304;58	-1	307;24	-	309;49	-2	312;15	-3	314;40		12;34	+1
299	301;10		303;34		306;58	-60	308;22		310;46		313;10		315;34	-1	12;36	
300	302;13	-1	304;35	-1	306;58	-2	309;20	-	311;38	+2	314; 5	-3	316;27	-2	12;38	

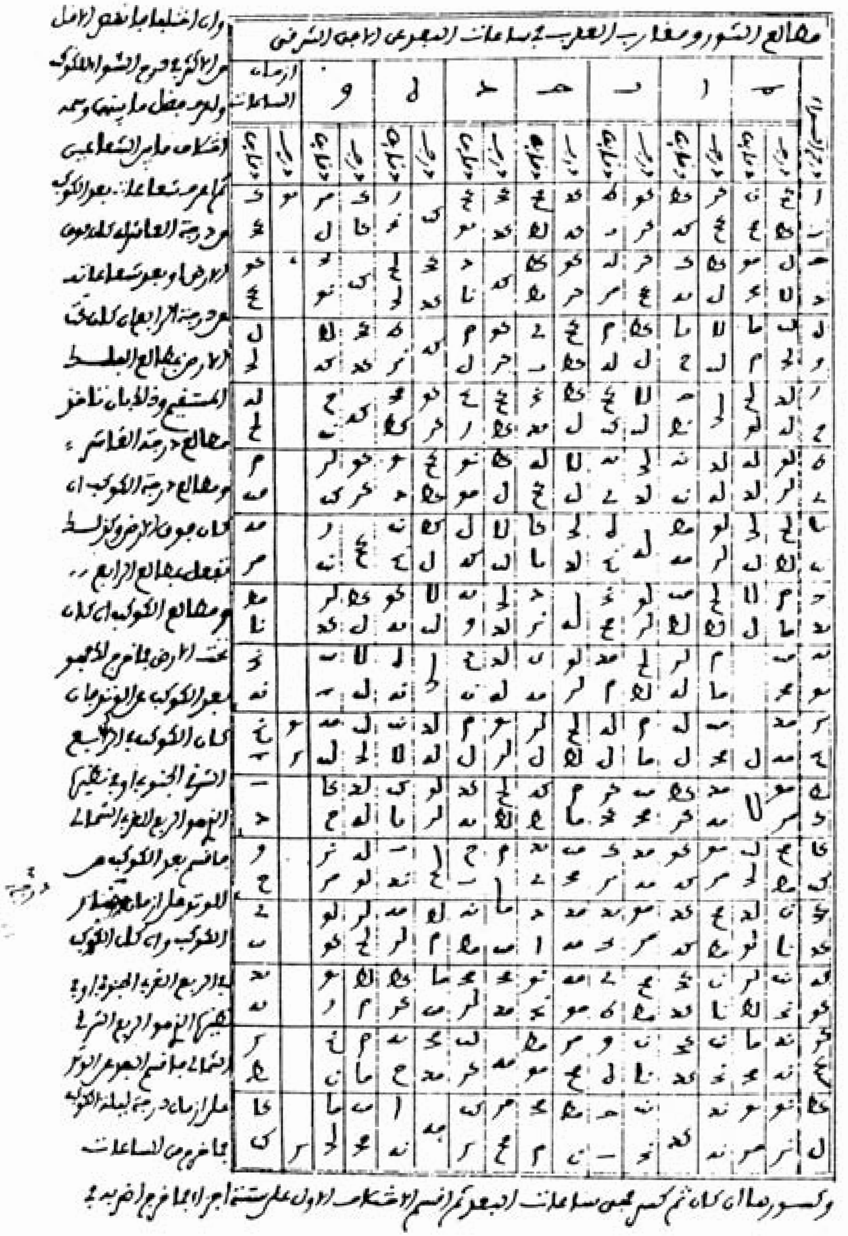
Ascensions of Aquarius and descensions of Leo
in hours of distance with respect to the eastern horizon
(مطالع الدلو ومغارب الأسد في ساعات البعد عن الأفق الشرقي)

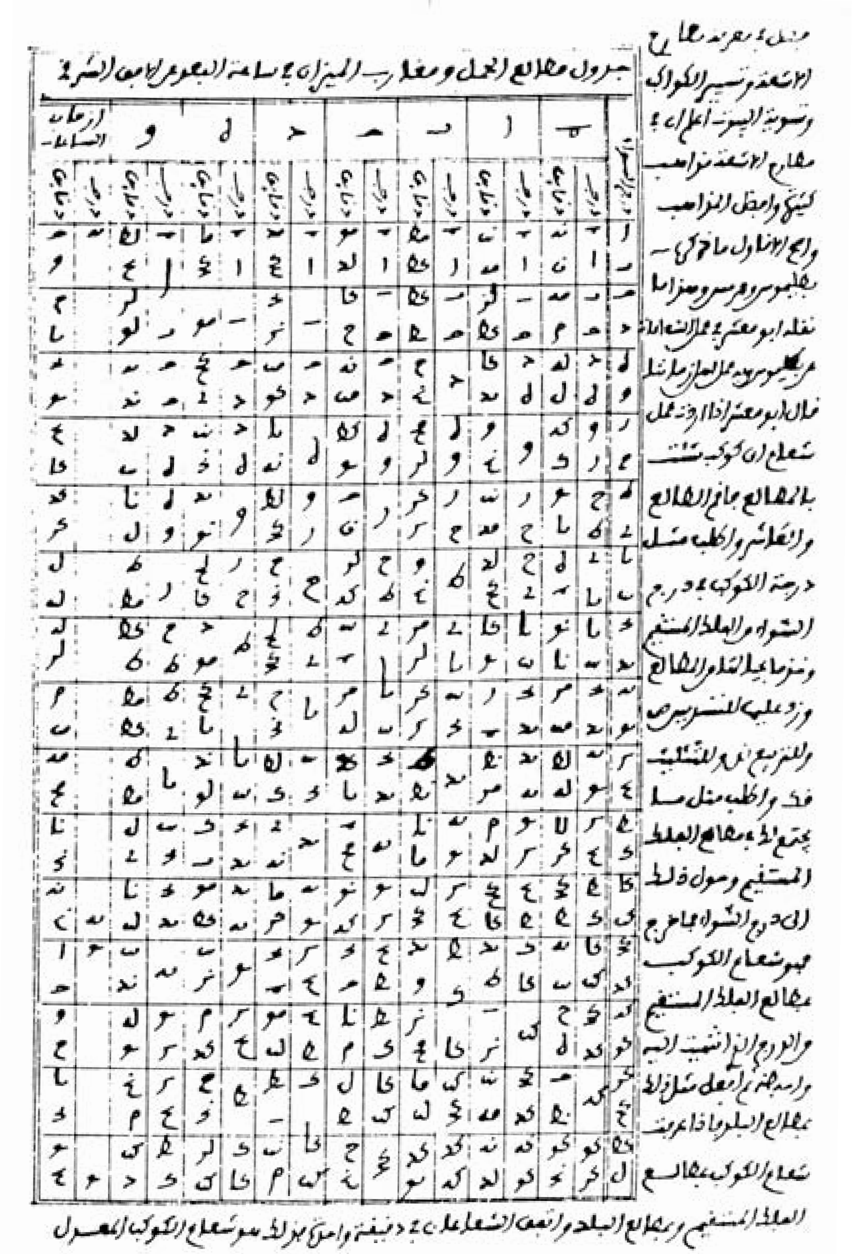
λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
301	303;15		305;56	-21	307;56	-1	310;17	-	312;37	-2	314;56	-1	317;19	-1	12;39	
302	304;17		306;56	-20	308;54	+1	311;13	+	313;32	+1	315;51	+1	318;10		12;41	
303	305;19		307;36		309;53		312;11	-	314;28	-1	316;45	-1	319; 2	-1	12;43	
304	306;21		308;36		310;52	-1	313; 7	-	315;23	-2	317;38	-2	319;53	-1	12;45	
305	307;23	-1	309;36		311;49	+1	314; 3	+	316;16	+2	318;30	+2	320;44	-1	12;47	
306	308;24		310;36	-1	312;47	-1	314;59	-	317;10	-2	319;22	-3	321;34	-1	12;48	
307	309;26	-1	311;36	-1	313;45		316;55	-6	318; 4	+1	320;14	+1	322;24	-1	12;50	
308	310;27	-1	312;36	-2	314;42		316; 7	+4	318;57	+1	321; 5	+1	323;13	-1	12;52	
309	311;28	-1	313;14	+19	315;40	-1	317;45		319;51		321;57		324; 3	-2	12;53	+1
310	312;29	-1	314;33	-1	316;37	-1	318;40		320;44		322;48		324;52	-2	12;56	
311	313;30	-1	315;32	-2	317;33	-2	319;35	-	321;36	-3	323;38	-4	325;39	-1	12;58	
312	314; 0	+29	316;30	-2	318;29	-2	320;29	-	322;28	-3	324;28	-4	326;28	-2	13; 0	
313	315; 0	+29	317;27		319;20	+5	321;22	+	323;20	+1	325;17	+2	327;15	-1	13; 2	+1
314	316; 0	+29	318;25	-1	320;20	-1	322;15	-	324;10	-1	326; 5	-1	328; 0	+1	13; 5	
315	317; 0	+29	319;22		321;15		323; 7	+	325; 0	+1	326;52	+2	328;45	+3	13; 6	+1
316	318;30	-1	320;21	-1	322;12	-1	324; 3	-	325;54	-1	327;45	-1	329;36	-1	13; 9	
317	319;29	-1	321;16	+1	323; 7	-1	324;56	-	326;45	-1	328;34	-1	330;23	-2	13;11	
318	320;28		322;15	-1	324; 1	-1	325;48	-	327;34	-2	329;21	-3	331; 8	-1	13;13	+1
319	321;27		323;11		324;46	+9	326;40	-	328;24	-1	330; 9	-2	331;53	-1	13;16	
320	322;26		324; 8		324;50	+60	327;32		329;14		330;56		332;38		13;18	
321	323;25		325; 5		326;44	+1	328;23	+	330; 3	+2	331;43	+2	333;22	+1	13;20	
322	324;24	-1	326; 1	-1	327;38	-1	329;14		330;51		332;28		334; 5	+2	13;22	+1
323	325;22		326;57		328;32		330; 7		331;42		333;17		334;52		13;25	
324	326;20		327;52	+1	329;24	+2	330;56	+	332;30	+2	334; 2	+3	335;35	+1	13;27	
325	327;19	-1	328;49	-1	330;20	-2	331;50	-	333;20	-2	334;51	-3	336;21	-1	13;30	
326	328;17	-1	329;45	-1	331;12		332;40		334; 8		335;56	-20	337; 4	-1	13;32	
327	329;14		330;35	+4	332; 5	-1	333;31	-	334;56	-2	336;20	-1	337;47	-1	13;34	+1
328	330;12		331;35		332;58		334;21		335;44		337; 7		338;30	-1	13;36	+1
329	331;10	-1	332;30		333;51		335;12		336;32	+1	337;53	+1	339;13	-1	13;39	
330	332; 7		333;25		334;44	-1	336; 2	-	337;20	-1	338;38	-1	339;56	-1	13;42	

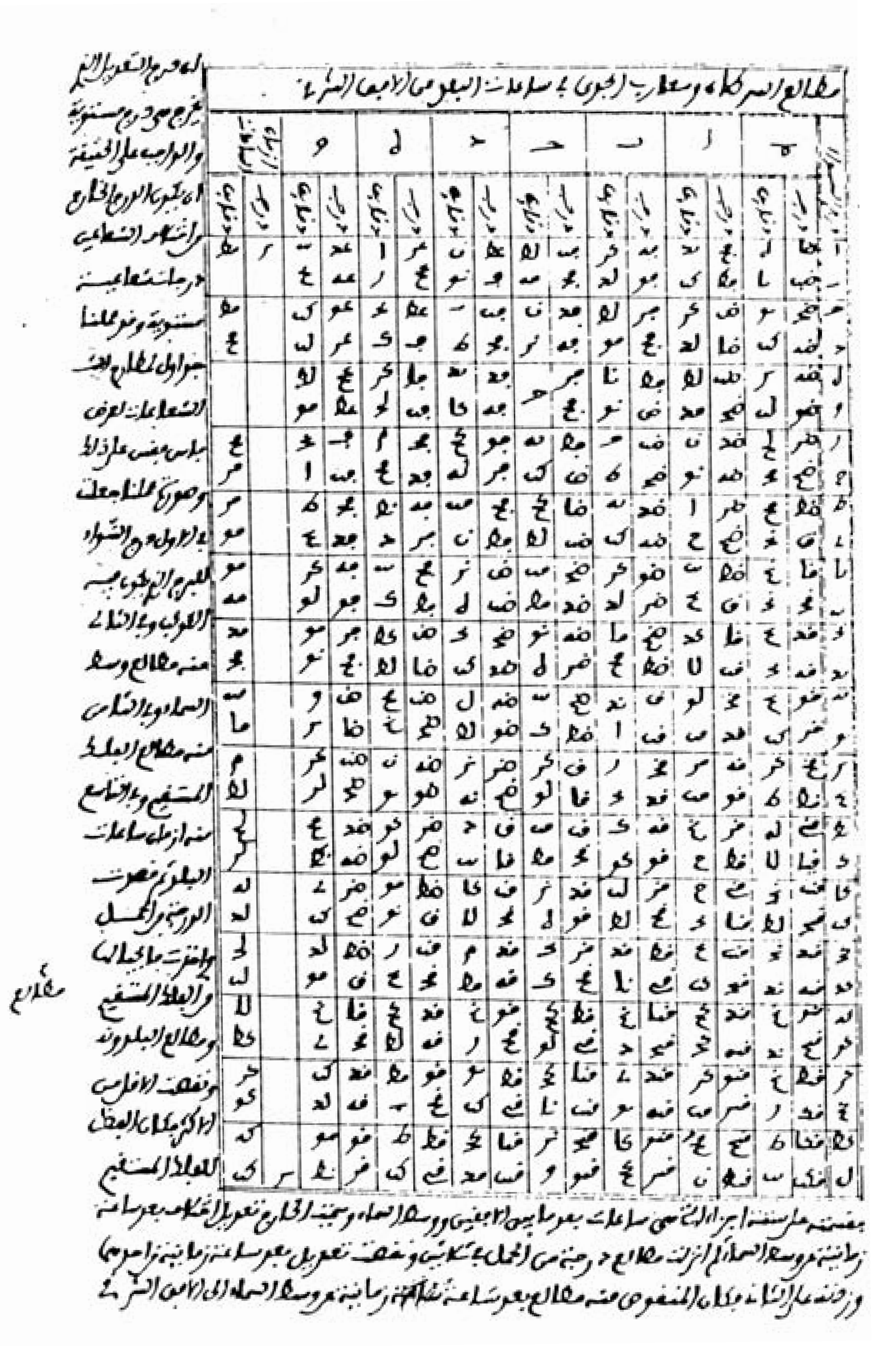
Ascensions of Pisces and descensions of Virgo
in hours of distance with respect to the eastern horizon
(مطالع الحوت ومغارب السنبلة في ساعات البعد عن الأفق الشرقي)

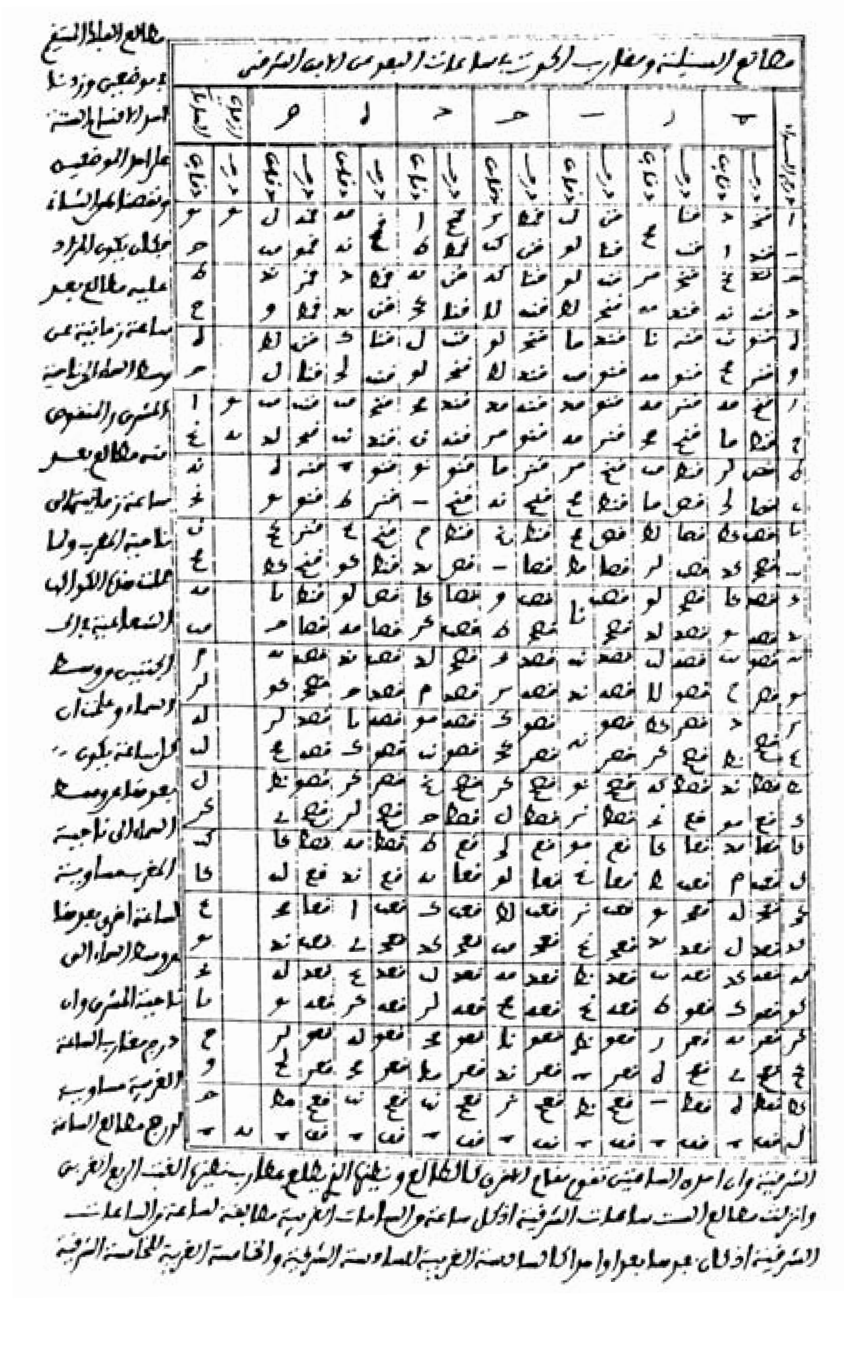
λ	hour 0	diff.	hour 1	diff.	hour 2	diff.	hour 3	diff.	hour 4	diff.	hour 5	diff.	hour 6	diff.	hour length	diff.
331	333; 4		334;20	-1	335;35	-1	336;51	-	338; 6	-2	339;25	-6	340;38	-1	13;44	
332	334; 1		335;14		336;28	-1	337;41	-	338;55	-2	340; 8	-2	341;21	-2	13;47	
333	334;28	+30	336; 9	-1	337;19	-1	338;30	-	339;40	-2	340;41	+7	342; 2	-1	13;49	
334	335;55		337; 3		338;11		339;20	-	340;28	-1	341;36	-1	342;44	-1	13;50	+2
335	336;52	-1	337;57	-1	339; 2	-1	340; 8	-	341;14	-3	342;20	-4	343;25	-1	13;54	
336	337;48		338;51		339;54		340;57		342; 0		343; 3		344; 6		13;57	
337	338;45	-1	339;45	-1	340;46	-2	341;47	-	342;17	+27	343;48	-4	344;48	-1	13;59	+1
338	339;41		340;39		341;37		342;34	+	343;32	+1	344;30	+1	345;38	-10	14; 2	
339	340;37		341;32		342;27		343;23	-	344;18	-1	345;13	-1	346; 8	+1	14; 5	
340	341;33		342;26		343;18	+1	345;18	-6	345; 4	+1	345;57	+1	346;50	-1	14; 7	
341	342;29		343;19		344; 9		345; 0	-	345;50	-1	346;40	-1	347;30		14; 9	+1
342	343;25		344;13		345; 0	+1	345;48	+	346;35	+2	347;23	+2	348;11	-1	14;12	
343	344;21		345; 6		345;51		346;36		347;21		348; 6		348;51	-1	14;15	
344	345;16		345;58		346;41	-1	347;24	-	348; 6	-2	348;49	-3	349;31	-1	14;18	
345	346;12		346;52		347;32		348;12		348;51	+1	349;31	+1	350;11	-1	14;20	
346	347; 8		347;45		348;23	-1	349; 0	-	349;37	-1	350;14	-1	350;51	-1	14;23	
347	348; 3		348;38		349;13		349;47	+	350;22	+1	350;56	+2	351;31	-1	14;25	+1
348	348;59	-1	349;31	-1	350; 3	-1	350;35	-	351; 7	-1	351;39	-1	352;11	-2	14;28	
349	349;54		350;23		350;53	-1	351;22	-	352;52	-62	352;21	-2	352;51	-2	14;32	-1
350	350;49		351;16	-1	351;42	-1	352; 5	+	352;56	-23	353; 3	-4	353;30	-2	14;33	
351	351;45	-1	352; 8		352;32		352;57	-	353;21	-1	353;45	-1	354; 9	-1	14;36	
352	352;40		353; 1		353;23	-1	353;44	-	354; 6	-2	354;27	-2	354;48	-1	14;39	
353	353;35		353;52	+1	354;10	+1	354;27	+	354;45	+2	355; 2	+3	355;26		14;42	-1
354	354;30		354;42	+4	355; 2		355;18		355;34		355;50		356; 6		14;44	
355	355;25		355;38		355;52	-1	356; 5	-	356;19	-2	356;30		356;45		14;47	
356	356;20		356;31		356;42		356;53		357; 3	+1	357;14	+1	357;25	-1	14;49	
357	357;15		357;23		357;31		357;39		357;47		357;55		358; 3		14;52	
358	358;10		358;16	-1	358;21	-1	358;26	-	358;32	-2	358;37	-2	358;42		14;57	-2
359	359; 5		359; 8		359;11		359;13	+	359;16	+1	359;18	+2	359;21		14;59	-2
360	360; 0		360; 0		360; 0		360; 0		360; 0		360; 0		360; 0		15; 0	

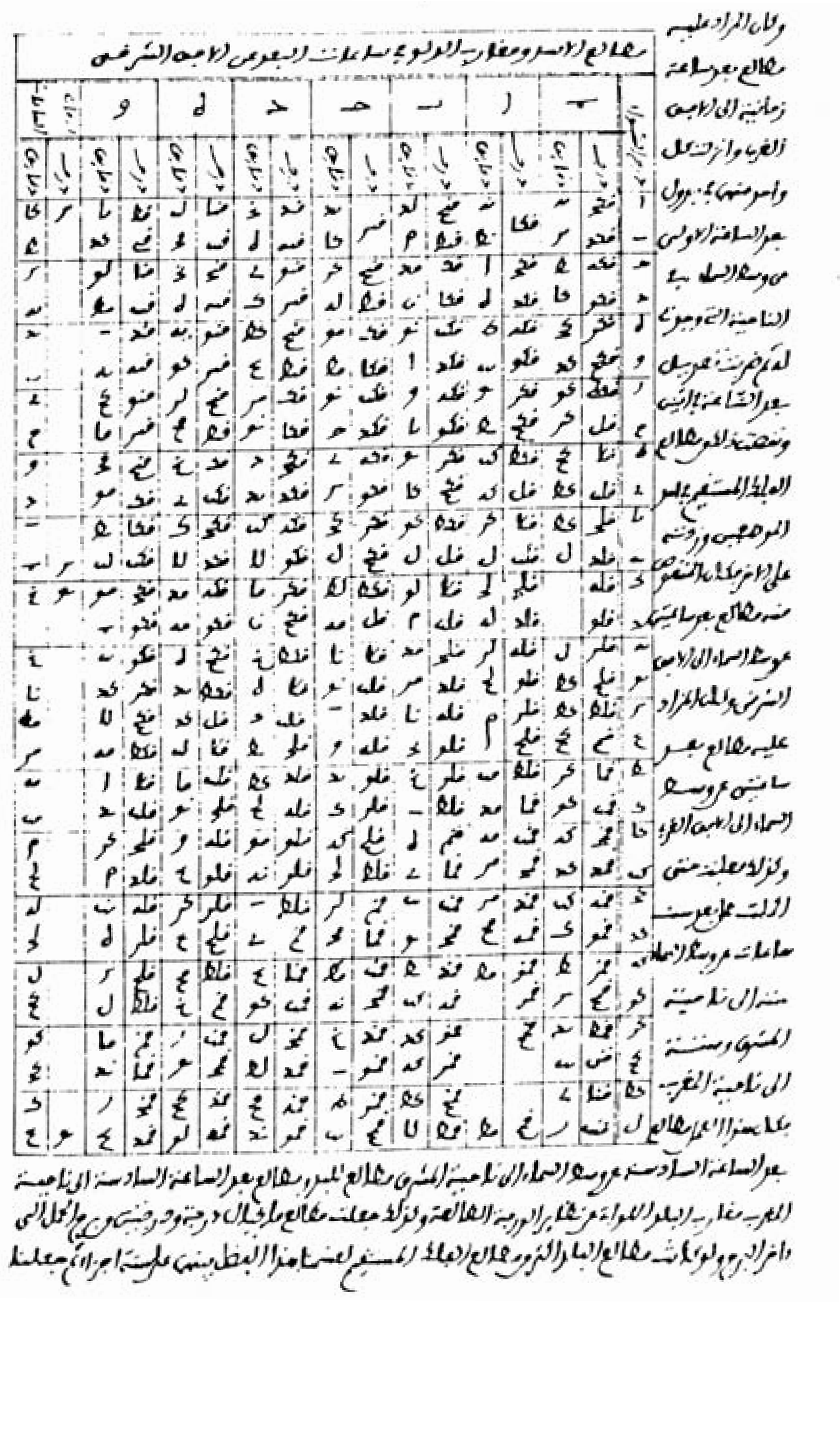
Appendix 4. Facsimile edition of pages 428-437 of MS D 2461

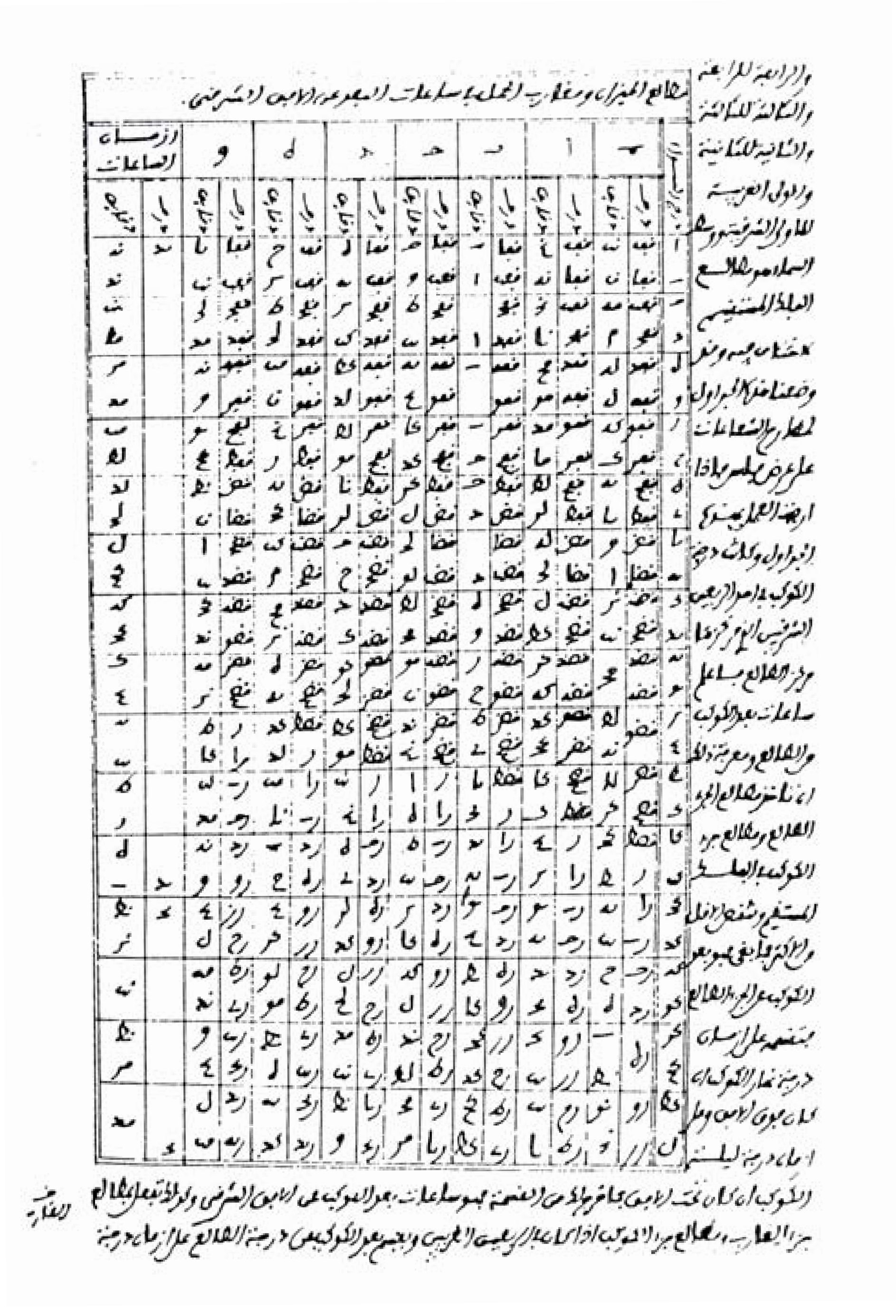








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للاولى الشرقية